Rings Domination Number of Sun Flower and Closed Sunflower Graphs

Rajan D[1], Jenila R.[2]
[1]*Assistant Professor, Arunachala HiTech Engineering College, Mullanganavilai, Marthandam*
[2]*Assistant Professor, Arunachala HiTech Engineering College, Mullanganavilai, Marthandam*

Abstract **: Let $G = (V, E)$ be a graph. A dominating set $D \subset V(G)$ in G is a rings domination set if each vertex $v \in V - D$ is adjacent to at least two vertices in $V - D$. If D is a rings dominating set, then D is called a minimal ring dominating set if it has no proper rings dominating set. A minimum rings dominating set is a rings dominating set of smallest size in a given graph. The minimum cardinality of all minimal rings dominating set, denoted by $\gamma_{ri}(G)$, is called the rings domination number. Here, we also obtain $\gamma_{ri}(G)$ for Sunflower graph and Closed Sunflower graph.**

Index Terms: **Dominating Set, Domination number, Rings dominating set, Rings domination number.**

I. INTRODUCTION:

All graphs considered in this paper are finite, undirected graphs and we follow standard definitions of graph theory as found in [4].

Let $G = (V, E)$ be a graph of order p. *The open neighborhood* of a vertex $v \in V(G)$ is $N(v) = \{u \in V(G) / uv \in E(G)\}$. The *closed neighborhood* of v is $N[v] = N(v) \cup \{v\}$. For a set $S \subseteq V$, the *open neighborhood of a set* $N(S)$ is defined to be $\bigcup_{v \in S} N(v)$, and the *closed neighborhood of a set* is $N[S] = N(S) \cup S$.

A set of points in G is *independent* if no two of them are adjacent. A *walk* in a graph is a sequence of alternating vertices and edges $v_1 e_1 v_2 e_2 \ldots v_n e_n v_{n+1}$ beginning and ending with vertices in which each edge is incident with the proceeding and succeeding vertices. A *path* is a walk with no repeated vertices. A path on n vertices is denoted by P_n. A *cycle* is a closed walk with all vertices are distinct except the starting and ending vertices. A cycle of order n is denoted by C_n. A *wheel graph* is formed by connecting a single vertex to all vertices of a cycle and is denoted by $W_n = K_1 + C_n$. A *Sunflower graph* SF_n is a graph obtained by replacing each edge of the rim of a wheel graph W_n by a triangle such that two triangles share a common vertex if and only if the corresponding edges in W_n are adjacent in W_n. A *Closed Sunflower graph* CSF_n is the graph obtained by joining the independent vertices of a sunflower graph SF_n which are not adjacent to its central vertex so that these vertices induce a cycle on n vertices.

A subset S of V is called a *dominating set* if every vertex in $V - S$ is adjacent to some vertex in S. A dominating set is *minimal dominating set* if no proper subset of S is a dominating set of G. The *domination number* $\gamma(G)$ is the minimum cardinality taken over all minimal dominating sets of G. A γ-set is any minimal dominating set with cardinality γ. A dominating set $D \subset V(G)$ in G is a *rings domination set* if each vertex $v \in V - D$ is adjacent to at least two vertices in $V - D$. If D is a rings dominating set, then D is called a *minimal ring dominating set* if it has no proper rings dominating set. A minimum rings dominating set is a rings dominating set of smallest size in a given graph. The minimum cardinality of all minimal rings dominating set, denoted by $\gamma_{ri}(G)$, is called the *rings domination number*.

II. MAIN RESULTS:

Theorem 2.1. For the Sunflower graph $SF_n, n \geq 3$, $\gamma_{ri}(SF_n) = n + 1$.

Proof. Let $V(SF_n) = \{v\} \cup \{u_i, v_i / i = 1,2, \ldots, n\}$, where v is the vertex in the Centre of the wheel, $u_i (1 \leq i \leq n)$ are the vertices on the cycle of the wheel and $v_i (1 \leq i \leq n)$ are the independent vertices which are not adjacent to u such that each $v_i(1 \leq i \leq$

$n-1)$ is adjacent to (u_i, u_{i+1}) and v_n is adjacent to (u_n, u_1).

Consider the dominating set $K = \{v\} \cup \{v_i / 1 \leq i \leq n\}$ and $V - K = \{u_i / 1 \leq i \leq n\}$. Here each vertex in $V - K$, that is, $u_1, u_i (2 \leq i \leq n-1)$ and u_n are adjacent to $\{u_2, u_n\}, \{u_{i-1}, u_{i+1}\}$ and $\{u_1, u_{n-1}\}$ respectively. So, each vertex in $V - K$ is adjacent to two vertices in $V - K$ and hence K is a ring dominating set. Therefore, $\gamma_{ri}(SF_n) = n + 1$.

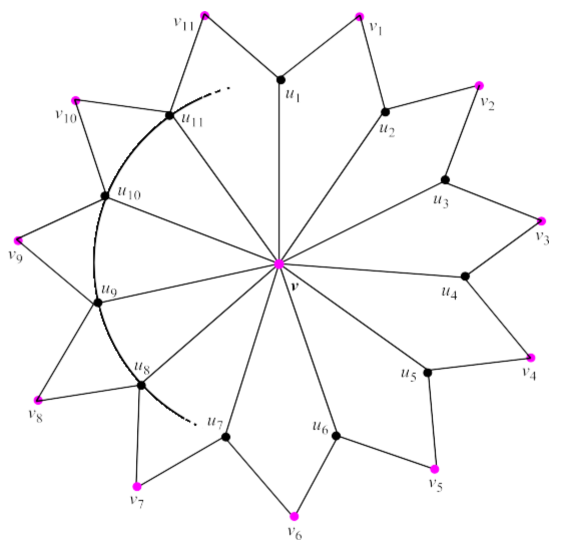


Figure 2.1

Theorem 2.2. The Closed Sunflower graph CSF_n has

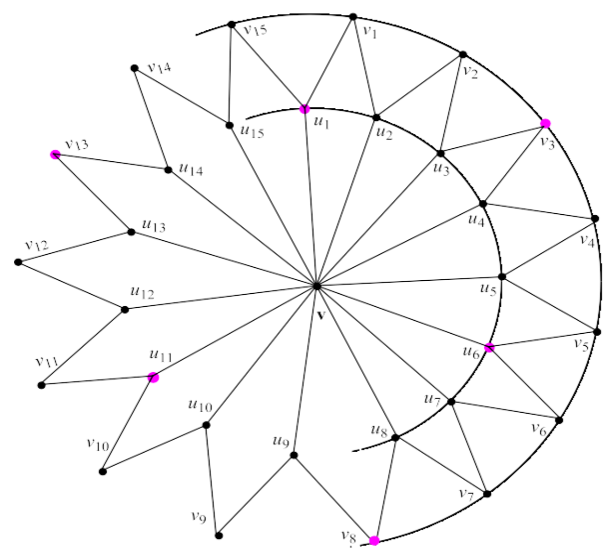
$$\gamma_{ri}(CSF_n) = \begin{cases} 2\left(\frac{n}{5}\right) & if\ n \equiv 0 (mod\ 5) \\ 2\left\lceil\frac{n}{5}\right\rceil - 1 & if\ n \equiv 1,2\ (mod\ 5), \\ 2\left\lceil\frac{n}{5}\right\rceil & if\ n \equiv 3,4 (mod\ 5) \end{cases}$$ for $n \geq 3$.

Proof.

Let $V(CSF_n) = \{v\} \cup \{u_i, v_i / i = 1,2, \ldots, n\}$, where v is the vertex in the Centre of the wheel and $u_i (1 \leq i \leq n)$ are vertices on the cycle of the wheel and $v_i (1 \leq i \leq n)$ are the vertices on the outer circle such that each $v_i (2 \leq i \leq n-1)$ is adjacent to $(u_i, u_{i+1}, v_{i-1}, v_{i+1})$ and v_1 and v_n are adjacent to (u_1, u_2, v_2, v_n) and (u_1, u_n, v_1, v_{n-1}) respectively. Suppose R is a $\gamma_{ri} -$ set. We consider the following five cases.

Case 1: $n \equiv 0 (mod\ 5)$. Consider the dominating set $D = \{u_{5i-4} / 1 \leq i \leq \frac{n}{5}\} \cup \{v_{5i-2} / 1 \leq i \leq \frac{n}{5}\}$ and $|D| = 2\left(\frac{n}{5}\right)$. Here each vertex in $V - D$ has degree 2 or 3 and so D is a rings dominating set. Since K is a γ_{ri} −set, $|D| \geq |K|$. Thus, $|D| = 2\left(\frac{n}{5}\right) \geq |K|$. On the other hand, since K is a γ_{ri} −set, then K must have at least $2\left(\frac{n}{5}\right)$ vertices in CSF_n. Thus, $|D| \geq 2\left(\frac{n}{5}\right)$. Therefore, $\gamma_{ri}(CSF_n) = |K| = 2\left(\frac{n}{5}\right)$.

Figure 2.2



Case 2: $n \equiv 1 (mod\ 5)$. Consider the dominating set $D = \{u_{5i-4} / 1 \leq i \leq \left\lceil\frac{n}{5}\right\rceil\} \cup \{v_{5i-2} / 1 \leq i \leq \left\lceil\frac{n}{5}\right\rceil - 1\}$ and $|D| = 2\left\lceil\frac{n}{5}\right\rceil - 1$. Here each vertex in $V - D$ has degree 2 or 3 and so D is a rings dominating set. Since K is a γ_{ri} −set, $|D| \geq |K|$. Thus, $|D| = 2\left\lceil\frac{n}{5}\right\rceil - 1 \geq |K|$. On the other hand, since K is a γ_{ri} −set, then K must have at least $\frac{n}{2}$ vertices in CSF_n. Thus, $|D| \geq 2\left\lceil\frac{n}{5}\right\rceil - 1$. Therefore, $\gamma_{ri}(CSF_n) = |K| = 2\left\lceil\frac{n}{5}\right\rceil - 1$.

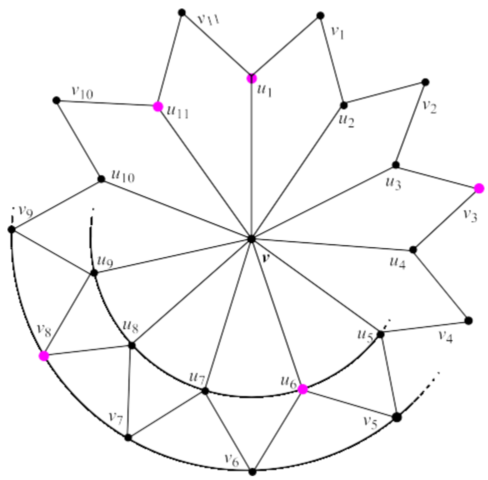


Figure 2.3

Case 3: $n \equiv 2 (mod\ 5)$.

Consider the dominating set $D = \{u_{5i-4} / 1 \le i \le \lceil \frac{n}{5} \rceil\} \cup \{v_{5i-2} / 1 \le i \le \lceil \frac{n}{5} \rceil - 1\}$ and $|D| = 2 \lceil \frac{n}{5} \rceil - 1$. Here each vertex in $V - D$ has degree 2 or 3 and so D is a rings dominating set. Since K is a γ_{ri} −set, $|D| \ge |K|$. Thus, $|D| = 2 \lceil \frac{n}{5} \rceil - 1 \ge |K|$. On the other hand, since K is a γ_{ri} −set, then K must have at least $2 \lceil \frac{n}{5} \rceil - 1$ vertices in CSF_n. Thus, $|D| \ge 2 \lceil \frac{n}{5} \rceil - 1$. Therefore, $\gamma_{ri}(CSF_n) = |K| = 2 \lceil \frac{n}{5} \rceil - 1$.

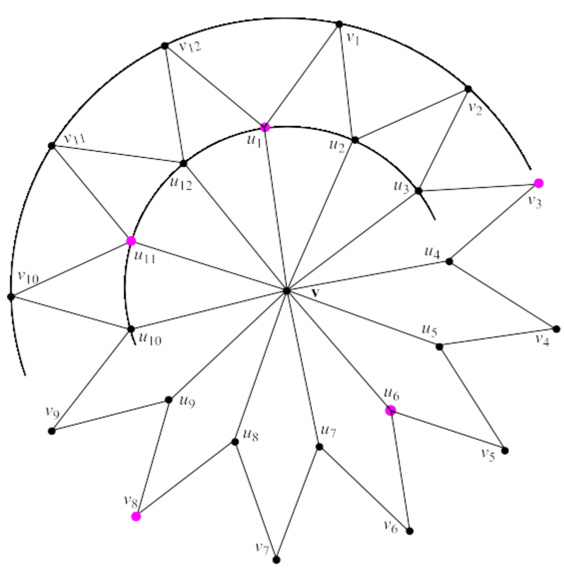


Figure 2.4

Case 4: $n \equiv 3 (mod\ 5)$.

Consider the dominating set $D = \{u_{5i-4} / 1 \le i \le \lceil \frac{n}{5} \rceil\} \cup \{v_{5i-2} / 1 \le i \le \lceil \frac{n}{5} \rceil\}$ and $|D| = 2 \lceil \frac{n}{5} \rceil$. Here each vertex in $V - D$ has degree 2 or 3 and so D is a rings dominating set. Since K is a γ_{ri} −set, $|D| \ge |K|$. Thus, $|D| = 2 \lceil \frac{n}{5} \rceil \ge |K|$. On the other hand, since K is a γ_{ri} −set, then K must have at least $2 \lceil \frac{n}{5} \rceil$ vertices in CSF_n. Thus, $|D| \ge 2 \lceil \frac{n}{5} \rceil$. Therefore, $\gamma_{ri}(CSF_n) = |K| = 2 \lceil \frac{n}{5} \rceil$.

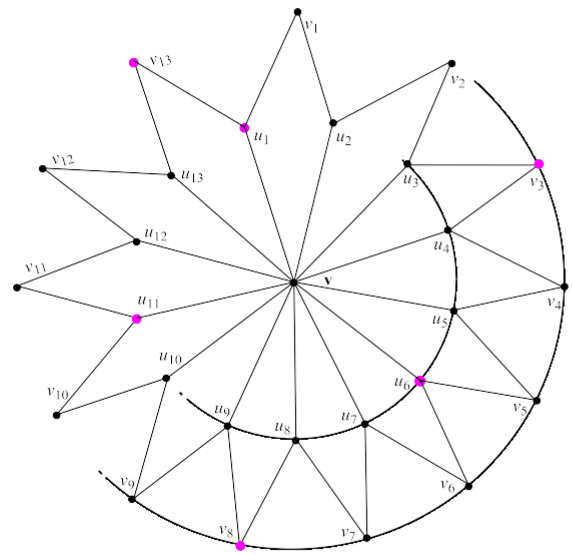


Figure 2.5

Case 5: $n \equiv 4 (mod\ 5)$.

Consider the dominating set $D = \{u_{5i-4} / 1 \le i \le \lceil \frac{n}{5} \rceil\} \cup \{v_{5i-2} / 1 \le i \le \lceil \frac{n}{5} \rceil\}$ and $|D| = 2 \lceil \frac{n}{5} \rceil$. Here each vertex in $V - D$ has degree 2 or 3 and so D is a rings dominating set. Since K is a γ_{ri} −set, $|D| \ge |K|$. Thus, $|D| = 2 \lceil \frac{n}{5} \rceil \ge |K|$. On the other hand, since K is a γ_{ri} −set, then K must have at least $2 \lceil \frac{n}{5} \rceil$ vertices in CSF_n. Thus, $|D| \ge 2 \lceil \frac{n}{5} \rceil$. Therefore, $\gamma_{ri}(CSF_n) = |K| = 2 \lceil \frac{n}{5} \rceil$.

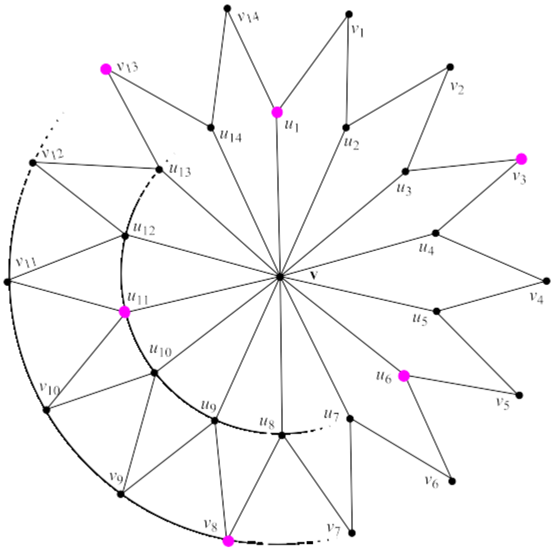


Figure 2.6

III. CONCLUSION

In this article, rings dominating sets in the sunflower and closed sunflower graphs are studied. Further, the rings domination number is also determined. Lastly, in the future, we plan to investigate the rings dominating set and rings domination number for a few unexplored graph families.

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