A REVIEW ON STEADY-STATE, ONE-DIMENSIONAL HEAT TRANSFER WITH THERMAL RESISTANCE NETWORK

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Abstract- **An understanding of the mechanisms of heat transferis becoming increasingly important in today's world. Conduction and convection heat transfer phenomena are found throughout virtually all of the physical world and the industrial domain. These two phenomena are very complicated and subjects to all bodies. To simplify their modeling for better understand of heat transfer, an analogy between thermal and electrical circuits are established. Assuming steady-state, one-dimensional heat transfer via conduction and/or convection modes, expressions are derived for thermal resistances across planar, cylindrical and spherical interfaces. Series, parallel and combination of series-parallel thermal network models are discussed (emphasizing similarity to electrical circuit theory). The equivalent thermal resistances are calculated for all thermal networks.**

Index Terms- **heat, conduction, convection, resistance, network**

I. INTRODUCTION

In engineering practice, an understanding of the mechanisms of heat transferis becoming increasingly important since heat transfer plays a crucial role in the design of vehicles, power plants, refrigerators, electronic devices, buildings, and bridges, among other things.Conduction and convection heat transfer phenomena are found throughout virtually all of the physical world and the industrial domain. The analytical description of these heat transfer mode are one of the best understood. Today conduction and convection heat transfer are still an active area of research and application. A great deal of interest has developed in recent years in topics like contact resistance, where a temperature difference develops between two solids that do not have perfect contact with each other.

For better understanding and simplificationthere can be introduced the network theory in heat transfer phenomena. A similarity between these two different mechanisms (heat transfer and charge transfer) suggests that a similar network model could also apply to thermal networks. These thermal networks would then be subject to the same principles of network theory as are electrical circuits, with the appropriate modification of what each variable represents (e.g. current as either charge flow or heat flow). Kirchhoff's laws for summing voltages about a loop or currents at a point would then equally apply to thermal circuits.

The goals of this paper are as follows:

- Introduction of concept of thermal network theory (for special cases of conduction and convection modes across planar, cylindrical and spherical geometry),
- Describe the analogy between thermal and electrical network,
- Derive appropriate expressions for thermal resistance,
- Describe the conditions for which series and parallel thermal models are appropriate, and
- Derive the expressions for equivalent resistance in all thermal networks for plane, cylindrical and spherical walls.

II. FUNDAMENTAL HEAT TRANSFER PRINCIPLES

Before deriving thermal resistance expressions, some fundamental principles of heat transfer will be briefly presented. It is assumed the reader has a qualitative understanding of conduction and convection heat transfer modes. In depth explanations of physical mechanisms by which these modes occur, as well as the validity limits of transfer expressions, are given in [1].

The primary forms of heat transfer are conduction, convection, and radiation. Within the scope of this paper, only conduction and convection are considered.

Fourier's law of heat conduction state that the rate of heat conduction through a plane layer is proportional to the temperature difference across the layer and the heat transfer area, but is inversely proportional to the thickness of the layer. The heat rate by this law is given by [1]

$$Q_{cond} = - k\, A \frac{dT}{dx}$$

Where the constant of proportionality k is the thermal conductivity of the material and dT/dx is the temperature gradient, which is the slope of the temperature curve on a T-x diagram

*Newton's law of cooling*stated that the rate of convection heat transfer rate is directly proportional to temperature difference between surface and environment and is given by[1]

$$Q_{conv} = h\, A_s\, (T_s - T_\infty)$$

Where h is the convection heat transfer coefficient in W/m^2,$A_s$is the surface area through which convection heat transfer takes place, $T_s$isthe surface temperature, and $T_\infty$is the temperature of the fluid sufficiently far from the surface.

Heat diffusion equation: The heat diffusion equation governs the time and spatial transfer of heat in a system. This "continuity equation" relates a medium's heat generation $\dot{g}$, net (inward) power transfer $\dot{E}_{in}$ - $\dot{E}_{out}$, and net rate of change of stored energy $\dot{E}_{stored}$ (with density □ and specific heat c_p) at a point according to [2]:

$$k\nabla^2 T + \dot{g} = \rho\, c\frac{\partial T}{\partial t}$$

Equation (3) can be understood in terms of an energy balance at an infinitesimal control volume: $\dot{E}_{\text{in}} - \dot{E}_{\text{out}} + \dot{E}_{\text{gen}} = \dot{E}_{\text{stored}}$.

III. SIMPLIFYING ASSUMPTIONS AND COORDINATE SETUPS

This section modifies the general form of (3) for specific flow conditions and geometries, producing spatial temperature distributions and heat rates.

(1)

Steady-state, source-free conditions: Neglecting temperature changes with time (steady state condition), we set $\partial T / \partial t = 0$ in equation (3). We furthermore restrict attention to media without heat sources (no heat generation) and thus set $\dot{g} = 0$. With these two restrictions, the heat diffusion equation reduces to:

$$k\, \nabla^2 T = 0$$

3.1. 1D heat transfer through planar wall

(2)

Assuming the Cartesian geometry of a planar wall (Fig. 1) in which the primary direction of heat flow occurs parallel to the x coordinate axis, we reduce the Laplacian of equation (4)

$$k\frac{d^2 T}{d^2 x} = 0 \qquad (5)$$

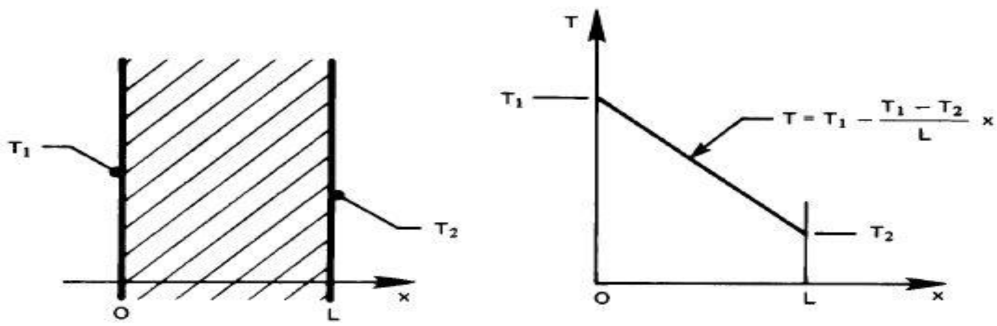

Figure 1: 1D heat conduction in plane wall

Assuming this wall has boundary conditions $T(x=0) = T_1$ and $T(x=L) = T_2$, we directly integrate equation (5) to produce a spatial temperature distribution according to [3]

$$\frac{T-T_1}{T_2-T_1} = \frac{x}{L}$$

This temperature distribution induces the following conductive heat flow [3]:

$$Q = kA\,\frac{\Delta T}{L}$$

3.2. 1D heat transfer through cylindrical wall

Assuming the cylindrical geometry of Fig. 2 (where only radial heat transfer occurs across the medium), equation (4)'s Laplacian takes the form

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r} \right) = 0$$

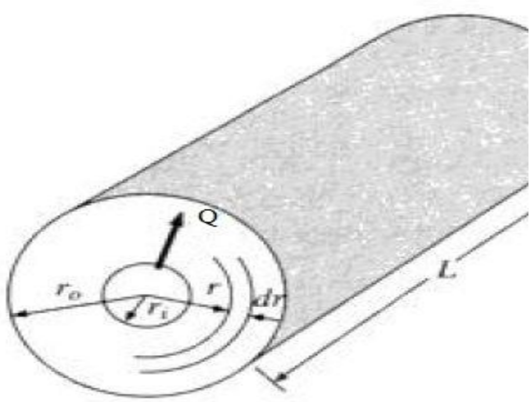

Figure 2: 1D Heat transfer through a cylinder (radial direction)

Directly integrating this expression subject to boundary conditions $T(r = r_i) = T_i$ and $T(r = r_0) = T_0$ yields the following spatial temperature profile [3]:

$$\frac{T-T_i}{T_0-T_i} = \frac{\ln (r/r_i)}{\ln (r_0-r_i)} \qquad (6)$$

This temperature variation gives rise to conductive heat flow through cylinder [3]

$$Q = \frac{2\pi kl\,\Delta T}{\ln (r_0/r_i)} \qquad (7)$$

3.3.1D heat transfer through spherical geometry

Spherical systems may also be treated as one-dimensional when the temperature is a function of radius only.

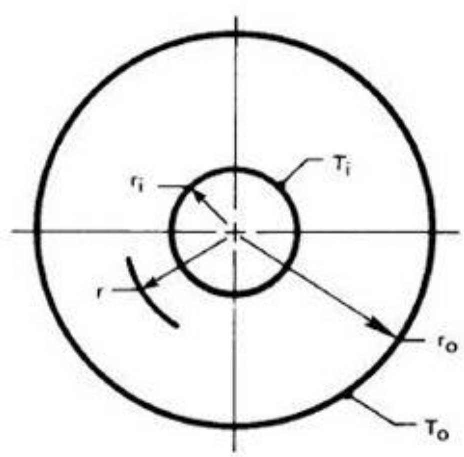

Figure 3: 1D heat transfer in sphere (radial direction)

Referring to fig 3 the heat flow through sphere is then given by [4]

$$Q = \frac{4\pi k \, (T_i - T_0)}{\frac{1}{r_i} - \frac{1}{r_0}} \qquad (11)$$

IV. USES OF THE THERMAL – ELECTRICAL ANALOGY

In the development of his law for electrical circuits, Georg Ohm performed experiments that modeled Fourier's lawof heat conduction. Consequently, an analogy between heat and electrical conduction can be observed.

Recall from circuit theory that resistance R_{elec} across an elementis defined as the ratio of electric potential difference ΔV across that element, to electric current *I* traveling through that element, according to Ohm's law,

$$R_{elec} = \frac{\Delta V}{I} \qquad (12)$$

Within the context of heat transfer, the respective analogues of electric potential and current are temperature difference ΔT and heat rate *Q*, respectively. Thus we can establish "thermal circuits" if we similarly establish thermal resistances *R* according to

$$R = \frac{\Delta T}{Q} \qquad (13)$$

We now consider specific expressions for *R* based on results for the three geometries specified in the previous section.

4.1.Expressions for Resistances

Planar wall conductive resistance: Again referring to Figure 1 and the result in equation (7), we see that thermal resistance may be obtained as

$$R_{x,cond} = \frac{\Delta T}{Q} = \frac{L}{kA} \qquad (14)$$

Cylindrical wall conductive resistance: Referring to Figure 2 and equation (10), we obtain conductive resistance through a cylindrical wall as

$$R_{r,cond} = \frac{\Delta T}{Q} = \frac{\ln{(r_0/r_i)}}{2\pi L k} \qquad (15)$$

Spherical wall conductive resistance: Referring to figure 3 and equation (11), we obtain conductive resistance through a spherical wall as

$$R_{r,cond} = \frac{\Delta T}{Q} = \frac{1}{4\pi k}\left(\frac{1}{r_i} - \frac{1}{r_0}\right) \qquad (16)$$

Convective resistance: The form of Newton's law of cooling, in equation (2), lends itself to a direct form of convective resistance, valid for either geometry.

$$R_{conv} = \frac{\Delta T}{Q} = \frac{1}{hA}$$

4.2.Series and Parallel Thermal Networks for plane wall

With expressions for calculating thermal resistances in hand, we move on to the important task of choosing appropriate models for thermal networks. The utility of thermal resistances exists in the ease with which otherwise complicated thermal systems are modeled. This section considers series and parallel thermal networks, drawing analogies to circuit theory's rules for equivalent resistance. For simplicity, only Cartesian geometries are considered.

4.2.1. Series networks:

Recall from circuit theory that resistors in series produce an equivalent resistance between input-output terminals that is the sum of individual resistances, owing to the fact that each individual resistor has the same current flowing through it.[5]

$$R_{series,eq} = \sum_i R_i \qquad (18)$$

Likewise, systems in which multiple elements are intercepted by a single heat flow line are modeled serially. Fig. 4 illustrates an example series model and circuit schematic for a double-exposed, layered window (note the presence of both conduction and convection transfer modes).

In this example, the equivalent thermal resistance would be the sum of each resistance shown, and would relate overall temperature difference across the network according to [6]

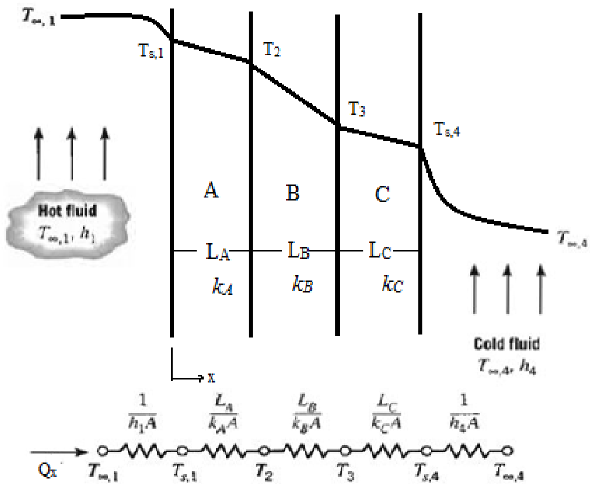

Figure 4: Equivalent thermal circuit for a series composite wall [6]

$$R_{eq} = \frac{T_{\infty,1}-T_{\infty,4}}{Q_x} = \frac{1}{A}\left(\frac{1}{h_1} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_2}\right) \quad (19)$$

4.2.2. Parallel networks:

Recall that electrical resistors in parallel produce equivalent resistance [5]

$R^{-1}_{parallel,eq} = \sum_i R^{-1}$

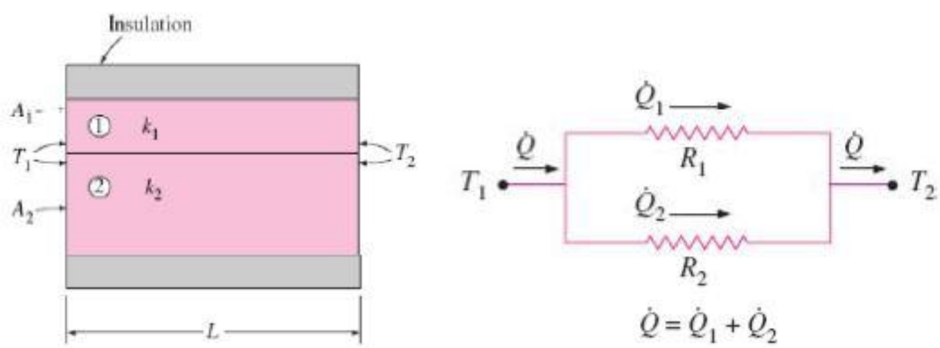

Figure 5: Equivalent resistance in parallel circuit

Such a model exists when multiple current paths exist between two nodes [7]. A similar situation occurs in thermal networks when two paths (through different media) exist between two points of the same temperature. An example system is shown in Fig. 5, with equivalent schematic. The equivalent resistance between the left and right faces is thus

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \quad (21)$$

4.2.3. Combined series–parallel networks

A combined series-parallel network is shown in fig.6 [8], the equivalent resistant of network can be calculated as

$$R_{eq} = R_{12} + R_3 + R_{conv} = \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_{conv} \quad (22)$$

Where, $R_1 = \frac{L_1}{k_1 A_1}$, $R_2 = \frac{L_2}{k_2 A_2}$, $R_3 = \frac{L_3}{k_3 A_3}$, $R_{conv} = \frac{1}{h A_1}$

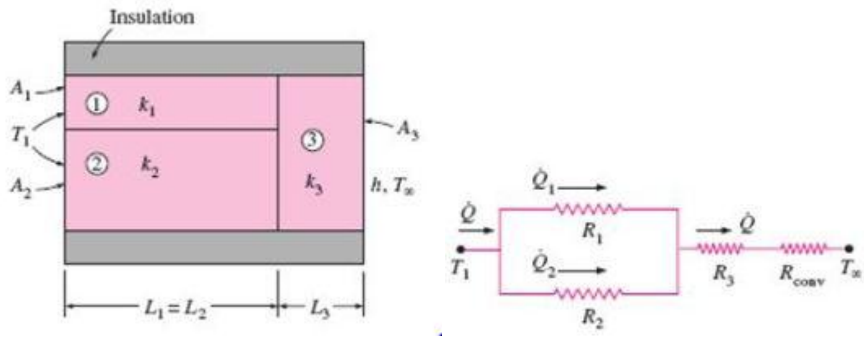

Figure 6: Equivalent resistance in combined series parallel network

4.3. Resistance network in cylindrical and spherical wall

The thermal resistance network for a cylindrical andspherical shell subjected to convection fromboth the inner and the outer sides is shown in fig 7 [8]. The equivalent resistance for cylindrical and spherical wall can be calculated as

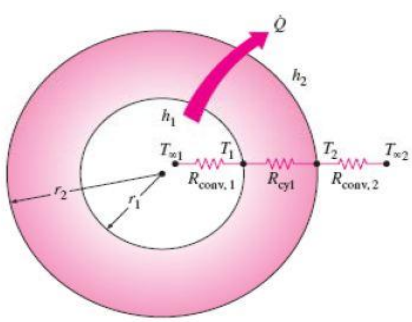

Figure 7: *Resistance network in cylindrical and spherical wall*

For cylinder,

$R_{eq} = R_{conv,1} + R_{cond,cyl} + R_{conv,2}$

$$= \frac{1}{2\pi r_1 L h_1} + \frac{\ln(r_2/r_1)}{2\pi L k} + \frac{1}{2\pi r_2 L h_2}$$

For sphere,

$R_{eq} = R_{conv,1} + R_{cond,sphere} + R_{conv,2}$

$$= \frac{1}{4\pi r_1^2 h_1} + \frac{r_2 - r_1}{4\pi r_1 r_2 k} + \frac{1}{4\pi r_2^2 h_2} \quad (24)$$

For multilayered cylindrical and spherical network as shown in figure 7, [8]

For multilayered cylinder,

$R_{eq} = R_{conv,1} + R_{cyl,1} + R_{cyl,2} + R_{cyl,3} + R_{conv,2}$

$$= \frac{1}{2\pi r_1 L h_1} + \frac{\ln(r_2/r_1)}{2\pi L k_1} + \frac{\ln(r_3/r_2)}{2\pi L k_2} + \frac{\ln(r_4/r_3)}{2\pi L k_3} + \frac{1}{2\pi r_2 L h_2}$$

For multilayered sphere,

$R_{eq} = R_{conv,1} + R_{sphere,1} + R_{sphere,2} + R_{sphere,3} + R_{conv,2}$

$$= \frac{1}{4\pi r_1^2 h_1} + \frac{r_2 - r_1}{4\pi r_1 r_2 k_1} + \frac{r_3 - r_2}{4\pi r_1 r_2 k_2} + \frac{r_4 - r_3}{4\pi r_1 r_2 k_3} + \frac{1}{4\pi r_2^2 h_2} \quad (26)$$

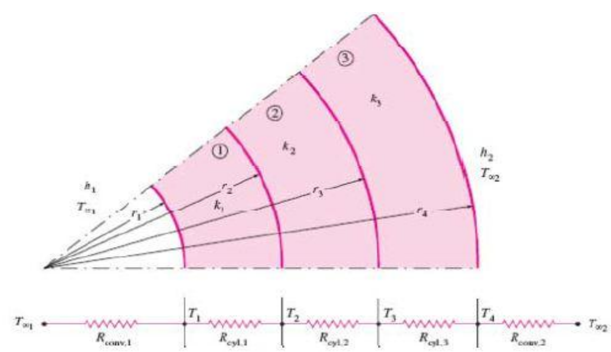

Figure 8: multilayered cylindrical wall

(23)

V. RESULTS AND DISCUSSIONS

The concepts of conductive and convective thermal resistance were derived from fundamental laws governing heat transfer and the heat diffusion equation. Three specific geometries (planar and cylindrical and spherical walls) were considered in obtaining explicit forms for conductive resistances. Thermal network modeling was then covered, emphasizing the conditions appropriate for series, parallel and combination of series-parallel networks. The analysis of thermal resistance network was also done for cylindrical and spherical walls considering conductive and convective resistance.

VI. CONCLUSIONS

In conclusion, we have successfully outlined the process of setting up and analyzing steady-state, 1D heat transfer problems for the geometries and modes indicated. The analytical capabilities provided in this discussion are deemed worthy tools for the system engineer, as system sensitivities (such as change in interface temperature for incremental changes in thicknesses, etc.) and overall system performance can now be achieved by recycling the analytical framework used in linear circuit theory.

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