Compressive Sensing based Image Reconstruction Algorithms – A Performance Review

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Abstract—Compressive Sensing (CS) is a powerful high resolution image modeling technique which has been successfully applied in digital image processing and various computer vision applications. This paper will portrait the current image reconstruction algorithms, their drawbacks and an overview of performance metrics such as image quality and PSNR improvements. The performance metric modelling is carried to trade-off choice of reconstruction algorithm with the quality of the reconstructed image considering various applications.

Index Terms—Compressive Sensing, Matching Pursuit, Basis Pursuit, Greedy algorithm, Sparse representation, Smoothed L0 norm

I. INTRODUCTION

In recent work, the problem of reconstructing time sequences of spatially sparse signals, with unknown and slow time varying sparsity patterns, from a limited number of linear “incoherent” measurements were studied and a solution called Compressed Sensing (CS) was proposed. Compressed Sensing (CS) has attracted considerable attention in areas of applied mathematics, computer science and electrical engineering by suggesting that it may be possible to surpass the traditional limits of sampling theory. CS builds upon the fundamental fact that we can represent many signals using only a few non-zero coefficients in a suitable basis or dictionary. Nonlinear optimization can then enable recovery of such signals from very few measurements. The CS technique is based on two fundamental principles: a) Sparse representation of the signal of interest in some basis, which is called the representation basis and b) Incoherence between the sensing matrix and the representation basis.

CS has been most valuable method that recovers the signals which are sparse or compressible based on some linear measurement matrix. The CS is being used to achieve accurate signal reconstruction while sampling a signal at low sampling rate, and much smaller than the traditional Nyquist theorem. Sparse representation is the representation of the signal with few number of elementary atoms. The CS encoding part is done in sparse representation step with the use of random measurement matrices while decoding part is taken care in the selection of appropriate reconstruction algorithm. The steps are shown in the following block diagram.

Figure 1: Block Diagram of Compressed Sensing

The rest of the paper is organized as follows. Section II outlines the details of CS based image reconstruction algorithms, Section III is the literature survey of the algorithms, Section IV briefly explains the applications. Experimental results and performance modeling based on PSNR are discussed in Section V and VI respectively, ending with conclusions in Section VII and References.

II. COMPRESSIVE SENSING BASED IMAGE RECONSTRUCTION ALGORITHMS

Compressive Sensing based image reconstruction algorithms can be classified into three types: 1)
Convex optimization algorithms, 2) Combinational algorithms 3) Greedy algorithms. In this paper, the focus will be given to perform literature survey on greedy and optimization algorithms. In the literature survey section, Matching Pursuit (MP), Orthogonal Matching Pursuit (OMP), Stagewise Orthogonal Matching Pursuit (StOMP), Compressed Sampling Matching Pursuit (CoSaMP), Gradient Pursuit (GP), Basis Pursuit (BP), Smoothed L0 norm(SL0), Improved SL0 (ISL0), Subspace pursuit (SP) methods would be explained briefly. The categorization is depicted in Figure 2.

Figure 2: Categorization of Image Reconstruction Algorithms

III. LITERATURE SURVEY

This section briefly outlines various reconstruction algorithms, with concise pseudo code along with pros and cons.

Matching Pursuit (MP)
Matching pursuit is a class of iterative algorithms that decomposes a signal into a linear expansion of functions that form a dictionary. At each iteration of the algorithm, matching pursuit chooses dictionary elements in a greedy fashion that best approximate the signal. Matching pursuit completely recovers the components that are explained by the dictionary elements. Because of the non-orthogonality, the results of each iteration may be suboptimal. Hence, it may take more iteration to obtain convergence [6].

Orthogonal Matching Pursuit (OMP)
Orthogonal matching pursuit (OMP) is an improvement on matching pursuit. The function is orthogonally projected onto all selected dictionary atoms, hence the term orthogonal matching pursuit. OMP is a greedy algorithm that can reliably recover a signal with m nonzero entries in dimension d given O (m ln d) random linear measurements of that signal. The OMP algorithm is faster and easier to implement, which makes it an attractive alternative to Basis Pursuit (BP) for signal recovery problems [2]. The OMP method is explained with pseudo code as it is used as a state of art method in most of applications for image reconstruction [3].

Algorithm (OMP for Signal Recovery)
Input:
- Measurement matrix \( \phi \)
- Measurement vector \( y = \phi x \)
- Sparsity level K

Output:
- Estimate \( \hat{x} \) to the signal \( x \)

Initialize:
Let the index set \( i = \emptyset \), the estimate \( \hat{x} = 0 \), and the residual \( r = y \). Repeat the following K \( \approx M / 2 \) times or until stopping condition holds

Identify:
Using the observation vector \( \hat{x} = \mathbf{D}^h y \), set \( J = \text{Max} \{ |\hat{x}_j| \} \),

Update:
Add the set \( j \) to the index set: \( i = i \cup j \), and update the residual:
\[
\hat{x}_i = (\mathbf{D}^h \phi)^{-1} \mathbf{D}^h y, \quad r = y - \phi \hat{x}_i
\]

Stopping Condition:
\[ \text{Norm}(r) < \delta \], where \( \delta \) is a small constant.

Stage-wise orthogonal matching pursuit (StOMP)
StOMP is an improvement on the OMP algorithm presented in the previous section. In contrast to OMP it allows multiple coefficients to be added to the model in a single iteration and runs for a fixed number of iterations. The choice of the thresholding parameter is inspired Gaussian noise removal, such as arising in digital communications. The process for choosing the threshold parameter is then governed by one of two strategies:
(a) False Alarm Control. The threshold is chosen so the false alarm rate does not exceed a specific per-iteration amount.
(b) False Discovery Control. The threshold is chosen so as not to exceed a certain fraction of the total
number of components added across all iterations [6], [7], [8].

Compressive Sampling Matching Pursuit (CoSaMP)
An extension to orthogonal matching pursuit algorithms is the CoSaMP (COMpressive Sampling Matching Pursuit) algorithm. The basis of the algorithm is OMP but CoSaMP, can be shown to have tighter bounds on its convergence and performance. The CoSaMP consists of five main steps as follows [6]-[8]:
(a) Identification: finds the largest 2s components of the signal. (b) Support Merge: merges the support of the signal proxy with the support of the solution from the previous iteration. (c) Estimation: estimates a solution via least squares with the constraint that the solution lies on a particular support. (d) Pruning: takes the solution estimate and compresses it to the required support. (e) Sample Update: updates the sample x so that it reflects the residual r that contains the signal that has not been approximated.

Gradient Pursuit (GP)
Gradient pursuit (GP) is yet another variation of matching pursuit. Instead of taking the update to simply be the scalar-product of the residual and dictionary element, the update occurs in a particular direction [6]. In matching pursuit and orthogonal matching pursuit, the update direction is taken to be in the direction of the element in the dictionary D that has largest inner product with the current residual. In OMP, once added, an atom will not be selected again as the process of orthogonalizing ensures that all future residuals will remain orthogonal to all currently selected atoms. In MP and GP however, orthogonality is not ensured. For this reason, the GP algorithm uses an update direction that may have already been used.

Basis Pursuit method (BP)
Basis Pursuit algorithm is one of the convex optimization algorithms to reconstruct the sparse signal. Interior point method [4] is used to solve the basis pursuit algorithm for sparse signal reconstruction. Basis pursuit algorithm is basically a technique to recover the original signal. The important advantages of basis pursuit over other algorithms are:
a) Prior sparsity knowledge of signal is not necessary,
b) Reconstruction problem can be formulated easily as linear programming problem which in turn is easy to solve and c) The performance of basis pursuit under noisy scenarios is good [5]. BP algorithm is a technique to solve l1 minimization problem which is formulated as linear programming problem for compressive sensing.

Smoothed L0 Norm method (SL0)
SL0 algorithm uses steepest descent method to approach the optimal solution. It does not require the sparsity as prior knowledge but there exist “notched effect” in search direction and the step-size is usually estimated with experiences [12]. Therefore, the computational performance of this method is still not efficient enough.

Improved Smoothed L0 Norm method (ISL0)
ISL0 algorithm uses modified Newton method to avoid the influence of “notched effect” as in SL0 method, and variable step-size Newton method is utilized to increase calculation speed and efficiency [12]. Small step-sizes will be applied for the minimization. For image reconstruction, ISL0 algorithm is firstly employed to recovery the measured value matrices. Then associate the high frequency sub-band coefficients with the preserved low frequency sub-band coefficients, and take on wavelet inverse transform to get the reconstructed image.

Subspace Pursuit method (SP)
SP has efficient reconstruction capability comparable to that of linear programming methods, and exhibits the low reconstruction complexity of matching pursuit techniques for very sparse signals. The algorithm can operate both in the noiseless and noisy regime, allowing for exact and approximate signal recovery, respectively [13]. It uses back tracing technique. In CS, the major challenge associated with sparse signal reconstruction is to identify in which subspace the measured signal lies. Once the correct subspace is determined, the non-zero signal coefficients are calculated by applying the pseudo inversion process.

IV. APPLICATIONS OF COMPRESSIVE SENSING BASED IMAGE RECONSTRUCTION
Compressive Sensing finds its applications in variety of fields like astronomy, biology, medicine, radar, seismology, to name a few. It is mainly used in areas like image processing for de-noising purpose and in
communication areas. The applications are discussed as below:

a. Cognitive Radio: In cognitive radio technology, wide band spectrum sensing is the problem of concern. To overcome this, compressive sensing was proposed [9]. It is assumed that received signal spectrum occupancy pattern is sparse and such wide spectrum can be reconstructed using basis pursuit algorithm. Further spectrum estimation techniques will determine which band is exactly vacant so that it can be used by another unlicensed user.

b. Signal de-noising: When the signal to be recovered in its entirety admits a sparse representation with respect to a known transform, then CS exploits this sparsity so as to fill in the missing data. If the data not only has missing samples but is also noisy, then CS de-noising should be used. The least square solution consists of merely filling in the missing data with zeros.

c. Data Separation: Assuming we are just given x, where \( x = x_1 + x_2 \), extracting \( x_1 \) and \( x_2 \) from x is a problem [10]. At first glance, this seems to be impossible, since there are two unknowns for one datum. To apply CS the approach is to select two orthonormal bases which are sparse. If the sparse vectors can be recovered then by mutual coherence \( x_1 \) and \( x_2 \) can be separately extracted which is beauty of CS.

d. Recovery of missing data: Unless additional data is given it is hard to recover ‘x’ from one of its sample ‘\( x_k \)’. To solve this in CS, a sparse orthonormal basis is selected then the 11-minimization is applied and solution can be provided by inpainting problem (term used in imaging science).

e. Ultra Wide Band (UWB) Signals: UWB communications is a promising technology for low power, high-bandwidth wireless communications. The advantages of UWB are: a) The implementation of the transmitter is simple because of the use of base-band signaling. b) UWB has little impact on other narrowband signals on the same frequency range, since its power spreads out on the broad frequency range. However, one of the challenges for UWB is that it requires extremely high sampling rate (several GHz) to digitize UWB signals based on the Nyquist rate, leading to high cost in hardware. Since UWB signals are sparse in the time domain, CS can be applied, which provides an effective solution to this problem by requiring much lower sampling rate [10].

V. EXPERIMENTAL RESULTS

This section explains the comparison of reconstruction image quality in-line different applications mentioned in section IV. The performance metric used to evaluate the reconstruction quality is the peak signal-to-noise-ratio (PSNR). The sparsity basis uses measurement values \( M = 50, 70, 90, \) Avg. The applications are compared with the ISL0, SP, OMP, CoSaMP methods. The results are plotted in the graph as shown below. For increased values of M, ISL0 method behaves more efficiently than other three methods. For the cognitive radio application, for \( M = 90 \) PSNR = 38.20 dB and average PSNR value is 35.76 dB. For \( M = 50 \) also good image quality is obtained but as the value of M increases better is the quality of reconstructed images with increased PSNR values. The reconstruction quality of SL0 algorithm has an average improvement of 0.84 to 1.95 dB wherein the ISL0 algorithm outweighs that of SL0 algorithm for about 0.75 to 1.27 dB in average.

VI. PERFORMANCE METRICS MODELING

This section presents the performance modelling of various reconstruction algorithms discussed in section III. Many performance metrics like PSNR, better vision quality, reconstructed image quality, convergence of an algorithm, imaging speed, Mean-square-error (MSE) etc. have to be considered while deciding choice of a particular reconstruction algorithm for an application in specific. Many applications of CS such as medical imaging (MRI, CT scans in particular), image de-noising in communication systems and spectrum band utilization, adding missing data, recovery of data, data
separation in channels bandwidth and so on demands varied quality of reconstruction algorithm. In order to design a better trade-off strategy in terms of matching particular reconstruction algorithm with specific application, performance modelling is carried out.

Figure 3: Need for Performance Modeling.

VII. CONCLUSION

With the proposed study of present challenges and performance modelling in-line with application, we would be proceeding with definition of a novel method to alleviate challenge of better reconstructed image quality and improve upon PSNR performance metric taking into consideration the relevant trade-offs with respect to various applications.

REFERENCES