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On 2-Domination Number of Some Graphs

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***Abstract*— Domination and 2 - domination numbers are defined only for graphs with non-isolated vertices. In a Graph G = (V, E) each vertex is said to dominate every vertex in its closed neighborhood. In a graph G, a subset S of V(G) is called a 2 - dominating set of G if every vertex in v ∈ V, is in V-S and has atleast two neighbors in S. The smallest cardinality of a 2 - dominating set of G is known as the 2 - domination number γ2(G). In this paper, we find 2 - dominating set of some special graphs and also find the 2 - domination number of graphs.**

***Index Terms*— Dominating set, 2 - Dominating Set, 2 - Domination Number**

I. INTRODUCTION

Fink and Jacobson introduced the concept of 2 – Domination Number [3]. Domination Number and 2 - Domination Number are defined only for graphs with non-isolated vertices. Every Graph with non -isolated vertex has a 2 - dominating set. In a graph G = (V,E), a subset D of V such that every vertex in V-D has a neighbor in D, such a set said to be a dominating set of G. The Dominating Number γ(G) is the minimum size of the dominating set of vertices in G. In a graph G = (V, E), subset S of V is a 2 - dominating set if every vertex v ∈ V, is in V – S has atleast two neighbors in S. The minimum cardinality of a 2 - dominating set of G is known as the 2 - domination number of γ2(G).

II.DEFINITIONS

2.1. Dominating Set
A set S ⊆ V of vertices in a graph G = (V, E) is called a dominating set if every vertex v ∈ V is either an element of S or is adjacent to an element of S.

2.2. Domination Number
The domination number of G, denoted by γ(G), is the minimum cardinality of a dominating set of G.

2.3. 2 Dominating Set
A dominating set S ⊆V(G) is said to be a 2 - dominating set if every vertex in V – S has atleast two adjacent vertices in S.

2.4. 2 - Domination Number
The minimum cardinality taken over all the minimal 2 - dominating set is called the 2 – domination number and it is denoted by γ2(G).

III.DEFINITIONS OF SOME SPECIAL GRAPHS

3.1. Flower Graph
A flower graph Fn is a graph obtained from a helm by adjoining each pendant vertex to the central vertex of the helm graph Hn.

3.2. Banana Tree Graph
A banana tree graph B (n, k) is a graph obtained by connecting one leaf of each of n – copies of an k –star with a single root vertex that is distinct from all the stars.

3.3. Coconut Tree Graph
A coconut tree graph CT (m, n) is a graph obtained from the path Pn by appending m new pendant edges at an end vertex of Pn.

3.4. Lollipop Graph
A lollipop graph L (m, n) is a special type of graph consisting of a complete graph Km on m vertices and a path graph Pn on n vertices connected with a bridge.

3.5. Tadpole Graph
A tadpole graph T (m, n) is a special graph consisting of a cycle graph Cm on m vertices and a path graph Pn on n vertices connected with a bridge.

IV. DOMINATION NUMBER OF SOME SPECIAL GRAPHS

Theorem 4.1

For any Flower Graph Fn (n ≥ 3), 2 - domination number is γ2(Fn) = n + 1

Proof

The flower graph has a universal vertex v connecting all the vertices of the cycle Cn having n vertices namely {x1,x2…, xn} and each node of the cycle adjoining a pendant edge. Let {y1, y2, ..., yn} be the pendant vertices joining to the central vertex v.

Here the central vertex v and either the pendant vertices or the vertices of the cycle are enough to dominate all the vertices of the graph Fn. Let us consider the dominating set of Fn = {v, x1, x2, ..., xn} or

{v, y1, y2, ..., yn}.

Thus, the 2 - domination number is γ2 is | {v, x1, x2, ..., xn} | or | {v, y1, y2, ..., yn} | Hence

γ2(Fn) = n + 1.

Theorem 4.2

For any Banana Tree Graph B (n, k), n ≥ 2 & k ≥ 3, 2 - domination number

γ2(B(n,k)) = n (k – 1)

Proof

A Banana tree B (n, k) is a graph obtained by connecting one leaf of each n copies of an k – star graph to a root vertex v.

Let the k – star graph has k – 1 leaf vertices and one internal node.

Here, the leaf vertices of the star graph are enough to dominate the graph B (n, k) for n times since we have n copies of k – star.

Thus, the 2 - domination number of a banana tree graph is γ2(B (n, k)) = n (k – 1).

Theorem 4.3

For a coconut tree graph CT (m, n), m, n ≥ 2,2 - domination number

$$\gamma2(\text{CT (m, n)}) = \begin{cases} \left(\frac{n+1}{2}\right) + m & for\ odd\ n \geq 3 \\ \frac{n}{2} + m & for\ even\ n \geq 2 \end{cases}$$

Proof.

Case (I)

For odd value of n ≥ 3

The coconut tree graph has m pendant vertices adjoining the path Pn to an end vertex. For odd value of n, choose the alternative vertex from the end vertex in the path graph and stop when all the alternative vertices are executed. Thus, we get $\left(\frac{n+1}{2}\right)$vertices. Here the m pendant vertices and the chosen$\left(\frac{n+1}{2}\right)$ vertices from the path graph are enough to dominate all the vertices of CT (m, n).

Hence the domination number is γ2CT (m, n) = $\left(\frac{n+1}{2}\right)$+ m

Case (II)

For even value of n ≥ 2

Choose the alternative vertices in the path graph starting from the end vertex and stop when all the alternative vertices are executed. Thus, we get $\frac{n}{2}$vertices. Here the m pendant vertices and the chosen$\frac{n}{2}$ vertices from the path graph are enough to dominate CT (m, n).

Hence the domination number is

γ2(CT (m, n)) = $\frac{n}{2}$+m.

Theorem 4.4

For a lollipop graph L (m, n), the domination number

For a lollipop graph L (m, n), the domination number

γ2(L (m, n))

$$= \begin{cases} \frac{m+n+2}{2} & for\ odd\ m \geq 3 for\ odd\ n \geq 3 \\ \frac{m+n}{2} & for\ even\ m \geq 4\ for\ even\ n \geq 2 \\ \frac{m+n+1}{2} & for\ odd\ m \geq 3 for\ odd\ n \geq 3 \\ & for\ odd\ m \geq 3 for\ odd\ n \geq 3 \end{cases}$$

Proof.

Case (I)

For odd value of n ≥ 3

For a path graph with odd value of n vertices. Choose the alternative vertices starting from the end vertex and stop until all the alternative vertices are executed. Thus, we get $\frac{m+1}{2}$vertices. Then from the complete graph of m vertices choose a vertex adjoin to the path graph and then choose any one of the vertices from the complete graph other than the chosen $\frac{m}{2}$vertex. Thus, we get 2 vertices. Thus, the chosen vertices dominate all the vertices of the graph L (m, n).

Hence the domination number γ2(L (m, n)) =

Case (II)

For even value of n ≥ 2

For a path graph with even value of n vertices. Choose an alternative vertex from the end vertex, continue like this and stop when all the alternative vertices are executed. Collect all such vertices we get vertices. Then from the complete graph choose a vertex adjoin to the path graph and then choose any one of the vertices the complete graph other than the chosen vertex. Thus, we get 2 vertices from the complete graph. Thus, the chosen vertices dominate all the vertices of the graph L (m, n).
Hence the domination number γ2(L (m, n)) =

Theorem 4.5
For a Tadpole graph T (m, n), the 2 - domination number
γ2(T (m, n)) =
Proof.
First consider the cycle graph Cm.
For odd value of m, choose a vertex that adjoins the path graph and then choose the alternative vertices in the cycle, continue like this and stop when all the alternative vertices are executed. Thus, we get vertices.
For even value of m, choose a vertex that adjoins the path graph and then choose the alternative vertices in the cycle, continue this process until all the alternative vertices are executed.
Thus, we get chosen vertices.
Next consider the path graph Pn.
For odd value of n, choose the end vertex and then choose the alternative vertices, continue like this and stop when all the alternative vertices are executed. Thus, we get vertices.
For even value of n, choose the end vertex and then choose the alternative vertices, continue this process and stop when all the alternative vertices are executed. Thus, we get chosen vertices.

Case i
For odd value of $m \geq 3$ & odd value of $n \geq 3$.
From the above algorithm, we get vertices and vertices i.e., vertices to dominate all the vertices of the graphs T (m, n). Hence the 2 - domination number
γ2(T (m, n)) =

Case ii
For even value of $m \geq 3$ & even value of $n \geq 2$.
From the above algorithm, we get vertices and vertices i.e., vertices to dominate all the vertices of the graph T (m, n). Hence the 2 - domination number,
γ2(T (m, n)) =

Case iii
For odd value of $m \geq 3$ & even value of $n \geq 2$
From the above algorithm, we get vertices and vertices i.e., vertices to dominate all the vertices of the graph T (m, n).
Hence the 2 - domination number
γ2 T (m, n) =

Case iv
For even value of $m \geq 4$ & odd value of $n \geq 3$.
From the above algorithm, we get vertices and vertices i.e., vertices to dominate all the vertices of the graph.
Hence the 2 - domination number is
γ2 (T (m, n)) =

CONCLUSION

In this paper, we have computed the 2 - dominating set and the 2 - domination number of some special graphs such as Flower graph, Banana tree, Coconut Tree, Lollipop Graph and Tadpole Graph.

REFERENCE

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