National Conference on Sustainable Engineering And Management (NCSEM24)
ISSN 2349-6002

# A Review of Isolated Domination Graph

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***Abstract*— A set *D* of vertices of a graph *G* is called a *dominating set* of *C* if every vertex in $V(G)$ - *D* is conterminous to a vertex in *D.* A dominating set S similar that the subgraph $(S)$ convinced by $S$ has at least one isolated vertex is called an *isolate dominating set.* An isolate dominating set none of whose proper subset is an isolate dominating set is a *minimal isolate dominating set.* The minimum and maximum cardinality of a minimal isolate dominating set are called *the isolate domination number* $\gamma_o$ and the *upper isolate domination number* $\Gamma_o$ respectively. In this paper we initiate a study on these parameters.**

***Index Terms*— Domination, isolate, Moments, Parallel Force**

## I. INTRODUCTION

This paper inmates a study on the parameters isolate domination number y, and the upper isolate domination number Γo. More specifically the exact values of γo and Γo for some commons, classes of graphs such as paths, cycles, wheels and complete multipartite graphs are determined in this paper. As an important result it is proved the parameters *γ0* and Γo got tit into the domination chain 1 and consequently an extended domination chain has been established. Further, some bounds for γo and Γo have been discussed in terms of order, size, degree and covering number.

Definition 1.1
A graph G consist of a pair (V(G), X(G)) where V(G) is a non-empty finite set whose elements are called points or vertices and X(G) is a set of unordered pairs of distinct elements of V(G). The elements of X(G) are called lines or edges of the graph

Definition 1.2
A graph G is called a bigraph or bipartite graph if V can be partitioned into two disjoint subsets $v_1$ and $v_2$ such that every line of G joins a point of $v_1$ to a point of $v_2$. ($v_1$, $v_2$) is called a bipartition of G

Definition 1.3
A dominating set S of a graph G is said to be an isolate dominating set of G if < S > has atleast one isolated vertex.

Definition 1.4
An isolate dominating set S is said to be a minimal isolate dominating set if no proper subset of S is an isolate dominating set.

Definition 1.5
The minimum and maximum cardinality of a minimal isolate domination number $\gamma_o$(G) and the upper isolate domination number $\Gamma_o$(G) respectively.

Definition 1.6
An isolate domination set of cardinality $\gamma_o$ is called a $\gamma_o$ set. Similarly, the sets γ - set, Γ set and $\Gamma_o$ set are defined.

Definition 1.7
A subset S of V is called an independent set of G if no two vertices of S art adjacent in G.

Theorem 1.1
A dominating set D is a minimal dominating set if and if only if for each vertex u in D, one of the following conditions holds.

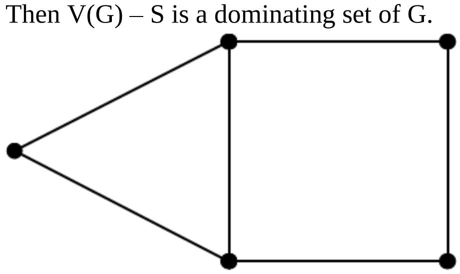
(i) u is an isolate of < D >.
(ii) There exist a vertex v in V – D, for which N(v) ∩ D = {u}.

Proof
Assume that D is a minimal dominating set of G.
Then for every vertex u∈D and D – {u} is not a dominating set.
Then there exists an vertex v in V(G)–(D–{u}) that is adjacent to no vertex of D–{u}.

If v = u then u is adjacent to no vertex of D.
Suppose v ≠ u.
Then the vertex v is adjacent to atleast one vertex of D.
Since D is a minimal dominating set and v not belongs to D.
Therefore v is adjacent to no vertex D – {u}.
N(v) ∩ D = {u}.
Conversely,
To prove D is a minimal dominating set.
Suppose D is not a minimal dominating set.
Then there exists a vertex u∈D such that D – {u} is a dominating set.

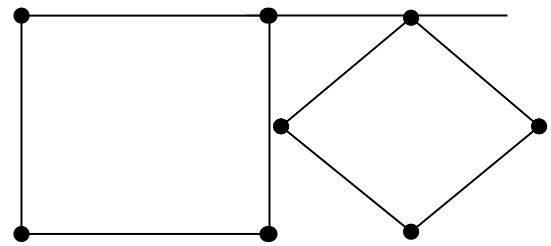
Hence u is adjacent to atleast one vertex in D – {u}.
Therefore condition (i) does not hold.
Also if D – {u} is a dominating set then every vertex V – D is adjacent to atleast one vertex in D – {u}.
Therefore (ii) does not hold for u.
Thus neither condition (i) nor (ii) holds.
This contradicts to our assumption.
Hence D is a minimal dominating set.

Theorem 1.2
If G is a graph with no isolated vertices, then the complement V – S of every minimal dominating set S is a dominating set.
Proof
Let v ∩ S.
Then v has atleast one of the properties of theorem 2.5.
Suppose that there exists a vertex w in V(G) – S such that N(w) ∩ S = {v}.
Hence v is adjacent to some vertex in V(G) – S.
Suppose that v is adjacent to no vertex in S.
Then v is an isolated vertex of the subgraph < S >.
Since v is not isolated in G.
The vertex v is adjacent to some vertex of V(G) – S.
Then V(G) – S is a dominating set of G.



Result 1.3
For any graph G of order n, Γ(G) + δ(G) ≤ n.

Example



{$v_1$, $v_2$, $v_5$}, {$v_8$, $v_3$, $v_5$} are dominating sets.
The minimal dominating set is {$v_8$, $v_3$, $v_5$} Γ(G) – maximum cardinality of the minimal dominating set.
δ(G) – Minimum degree of G
n – number of vertices.
δ(G) = 2
Γ(G) = 3
n = 8
Therefore, Γ(G) + δ(G) = 3 + 2
= 5
≤ n
Hence Γ(G) + δ(G) ≤ n.
Here we are going to determine the value of isolate domination number and the upper isolate domination number for some standard graphs such as paths, cycles, complete multipartite graphs and wheels.

Proposition 1.4
(i) For the paths $P_n$ and the cycle $C_n$, we have $\gamma_o(P_n) = \gamma_o(C_n) = \left\lceil \frac{n}{3} \right\rceil$,
(ii) $\Gamma_o(P_n) = \left\lceil \frac{n}{2} \right\rceil$ and $\Gamma_o(C_n) = \left\lfloor \frac{n}{2} \right\rfloor$
(iii) For a complete K-partite graph $G = K_{m_1, m_2, \ldots m_k}$,
$\gamma_o(G) = Min\{m_1, m_2 \ldots m_k\}$ and $\Gamma_o(G) = Max\{m_1, m_2 \ldots m_k\}$.
In particular $\gamma_o(K_n) = \Gamma_o(K_n) = 1$.
(iv) For the wheel $W_n$ on n vertices, $\gamma_o(W_n) = 1$ and $\Gamma_o(W_n) = \left\lfloor \frac{n-1}{2} \right\rfloor$.

Proof
(i) Obviously $\gamma_o(P_4) = 2$.
When n ≠ 4
any γ - set of $P_n$ is an isolate dominating set.
Therefore $\gamma_o(P_n) \leq \gamma(P_n)$.
Also we have $\gamma_o(P_n) = \gamma(P_n)$.
Therefore $\gamma_o(P_n) = \left\lceil \frac{n}{3} \right\rceil$ as $\gamma(P_n) = \left\lceil \frac{n}{3} \right\rceil$

now, if $P_n = \{v_1, v_2, \ldots\ v_n\}$

Then the set $S = \{v_{2i-1} / 1 \leq i \leq \lceil \frac{n}{3} \rceil\}$ is a minimal isolate dominating set.

Therefore $\Gamma_o(P_n) \geq \lceil \frac{n}{3} \rceil$.

Any set with more than $\lceil \frac{n}{3} \rceil$ vertices of $P_n$ can no longer be a minimal isolate dominating set.

We have $\Gamma_o(P_n) = \lceil \frac{n}{3} \rceil$.

In this similar way, we get

$\gamma_o(C_n) = \lceil \frac{n}{3} \rceil$ and $\Gamma_o(C_n) = \lfloor \frac{n}{2} \rfloor$.

(ii) Let $G = K_{m_1, m_2, \ldots, m_k}$ be a complete K-partite graph.

Obviously the K-parts of G are the only minimal isolate dominating sets of G.

Therefore, $\gamma_o(G) = \text{Min}\{m_1, m_2 \ldots m_k\}$ and $\Gamma_o(G) = \text{Max}\{m_1, m_2 \ldots m_k\}$.

In particular $\gamma_o(K_n) = \Gamma_o(K_n) = 1$.

(iii) Consider the wheel $W_n$, on n vertices. We know that the centre of $W_n$ dominates all other vertices.

Therefore $\gamma_o(W_n) = 1$.

Also, the maximum cardinality of the minimal isolate dominating set of $W_n$ is $(n – 1)$ vertices.

ie) $\Gamma_o(W_n)$ is the cycle on $(n – 1)$ vertices.

Therefore $\Gamma_o(W_n) = \Gamma_o(C_{n-1})$

By result (i), $\Gamma_o(C_n) = \lfloor \frac{n}{2} \rfloor$

$\Gamma_o(C_{n-1}) = \lfloor \frac{n-1}{2} \rfloor$

Hence $\Gamma_o(W_n) = \lfloor \frac{n-1}{2} \rfloor$

Proposition 1.5

If G is a disconnected graph with components $G_1, G_2, \ldots G_r$, then

a) $\gamma_o(G) = \min \{t_i\}, 1 \leq i \leq r$.

Where $t_i = \gamma_o(G_i) + \sum_{j=1, j\neq i}^{r} \gamma(G_j)$

b) $\Gamma_o(G) = \max \{S_i\}, 1 \leq i \leq r$.

Where $S_i = \Gamma_o(G_i) + \sum_{j=1, j\neq i}^{r} \Gamma(G_j)$

Proof

a) Assume $t_1 = \min \{t_1, t_2, \ldots, t_r\}$

Let S be a $\gamma_o$ set of $G_1$ and

Let $D_i$ be a $\gamma$ - set of $G_i$ for all $i \geq 2$.

Then the set $S \cup \left( \bigcup_{i=2}^{r} D_i \right)$ is an isolate dominating set of G.

Therefore $\gamma_o(G) \leq \gamma_o(G_1) + \sum_{j=2}^{r} \gamma(G_j)$

$= t_1$

$= \min \{t_i\}, 1 \leq i \leq r.$

$\gamma_o(G) \leq \min \{t_i\}, 1 \leq i \leq r$ -------(1)

Let S be a minimal isolate dominating set of G.

Then S must intersect the vertex set $V(G_i)$ of each component $G_i$.

Therefore $S \cap V(G_i)$ is a minimal dominating set of $G_i$, for all i = 1 to r.

Further, atleast one of the sets of $S \cap V(G_i)$ say $S \cap V(G_j)$, must be an isolate dominating set of $G_j$ for all $i \neq j$.

Therefore, $|S| \geq \gamma_o(G_j) + \sum_{i=1, i\neq j}^{r} \gamma(G_j) = t_j$

$= \min \{t_j\}, 1 \leq i \leq r.$

Therefore, $|S| \geq \min \{t_i\}, 1 \leq i \leq r$.

Which implies $\gamma_o(G) \geq \min \{t_i\}, 1 \leq i \leq r$ ……(2)

Since $|S| = \gamma_o(G)$

From equations (1) and (2),

$\gamma_o(G) = \min \{t_i\}, 1 \leq i \leq r.$

(b)For any value of i, a $\Gamma_o$ – set of $G_i$ together with the set $\bigcup_{j=1\ j\neq i}^{r} D_j$, where $D_j$ is a $\Gamma$ set of $G_j$, forms a minimal isolate dominating set of G.

Therefore, $\Gamma_o(G) \geq \max \{\Gamma_o(G_i) + \sum_{j=1, j\neq i}^{r} (G_j)\}, 1 \leq i \leq r$.

$\Gamma_o(G) \geq \max \{S_i\}, 1 \leq i \leq r$------------------- (3)

Let S be any minimal isolate dominating set of G.

Then $S \cap V(G_i)$ is a minimal isolate dominating set of $G_i$ for all i.

In particular, $S \cap V(G_i)$ must be a minimal isolate dominating set for atleast one value of i say j.

Then $|S| \leq \Gamma_o(G_i) + \sum_{i=1, j\neq i}^{r} (G_i)$

$= S_j$

$| S | \leq \max \{S_i\}, 1 \leq i \leq r.$

Therefore $\Gamma_o(G) \leq \max \{S_i\}, 1 \leq i \leq r$-------------- (4)

From equations (3) and (4)

$$\Gamma_o(G) = \max \{S_i\}, 1 \leq i \leq r$$

$$\text{Where, } S_i = \Gamma_o(G_i) + \sum_{j=1, j \neq i}^{r} (G_j) .$$

Result 2.10

Every independent dominating set in a graph G is an isolate dominating set. So that every graph possess an isolate dominating set as every graph has an independent dominating set.

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