Dynamics of magnetic field in a fluid with large electrical conductivity

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***Abstract* - This paper reviews the underlying physics of the dynamics of magnetic field in a fluid with large electrical conductivity. The magnetic field is highly non-dissipative in a a highly conducting fluid at large length scales and at the same time the field is highly dissipative at small length scales. The dynamics of magnetic field in between these two scales shows ubiquitous phenomena like generation of magnetic discontinuity and topological rearrangements of magnetic field lines. Such diverse dynamics of magnetic field then provides an opportunity to understand the real physical systems like solar corona , interplanetary medium, laboratory plasmas, etc.**

***Index Terms* - Magnetic topology, Magnetic discontinuity, Magnetic reconnection, Solar corona.**

I.INTRODUCTION

At very high temperature, the outermost electrons of gaseous atoms get knocked out from the atoms and a fully ionized gaseous medium is created. Such an ionized gaseous medium has a large electrical conductivity and shows plasma behavior [1]. The dynamics of magnetic field embedded in a highly conducting medium is governed by the magnetohydrodynamic (MHD) equations [2]. An important parameter in magnetohydrodynamics is the magnetic Reynolds number which is the ratio of diffusion time scale and the convective time scale [1]. The Reynolds number decides the relative dominance of the two scales. A high value of Reynolds number means the convection is dominant whereas a low value of Reynolds number means the diffusion is dominant. A fluid with infinite Reynolds number is governed by ideal MHD equations. An important property of ideal magnetohydrodynamics is that magnetic field lines are frozen to the fluid elements/parcels--- called frozen-in or flux-freezing condition [3]. Under the frozen-in condition, once we identify a magnetic field line being tied to a particular fluid element then this identity is maintained throughout the evolution of the fluid, i.e. the field line will remain intact to that particular fluid element during the course of fluid motion. In a broad sense any interlinking or connectivity of magnetic field lines--- called the magnetic topology--- is preserved. As the magnetic field lines are co-moving with the fluid elements and therefore such a system is termed as magnetofluid. Consider any two fluid elements which are approaching towards each other under the influence of some unbalanced forces like pressure, Lorentz force, etc. As two such fluid elements come into a direct contact with each other then at the surface of contact the magnetic field lines associated with respective fluid elements will also come into a direct contact with each other. Note that the magnetic field lines from the two fluid elements do not mix with each other because of the frozen-in condition. As such, at the surface of contact the magnetic field lines will develop magnetic discontinuity (MD) [3]. Because of such discontinuity in magnetic field, a large gradient in magnetic field develops across the common surface of contact, resulting in a high value of electric current density. This current density is totally confined to a two-dimensional surface and hence also called a current sheet (CS). In case of ideal magnetohydrodynamics, a true discontinuity in magnetic field appears where the thickness of the current sheet is zero. However, in real magnetofluid, the electrical conductivity is not infinite and there is some finite electrical resistivity. In case of real magnetofluid, the magnetic field lines associated with the two fluid elements do not come into a direct contact with each other, but rather, the diffusion becomes dominant as soon as the separation between the two fluid elements becomes very small where the value of magnetic Reynolds number is low. In such a scenario, a current sheet of finite thickness is developed in between the two fluid elements. Also, such current sheet will decay by the presence of very small but finite electrical resistivity--- resulting in generation of heat and kinetic energy along with a change in magnetic field topology. This process of energy conversion along with a change in field topology is called magnetic reconnection (MR) [1]. Such ubiquitous process of the generation of magnetic discontinuities and their eventual decay by magnetic reconnection plays a major role in heating the outer atmosphere of the Sun, i.e. the solar corona, at its million degree Kelvin temperature [3].

II. GENERATION OF MAGNETIC DISCONTINUITY: THEORETICAL BACKGROUND

The theory of magnetic discontinuity was first proposed by Parker [3]. According to Parker theory, when any continuous magnetic field with arbitrary magnetic topology relaxes towards the static equilibrium then it will develop magnetic discontinuity at the end state of equilibrium. An important property of static equilibrium is that the torsion coefficient is constant along the magnetic field line. However, a magnetic field with arbitrary topology may not have the torsion coefficient constant along the field lines. Because of this, an arbitrary field topology will develop magnetic discontinuity in the static equilibrium in order to accommodate the constant torsion coefficient of field lines. Mathematically, the generation of magnetic discontinuity can be also understood as follows. The equation governing the magnetostatic equilibrium is basically a nonlinear partial differental equation of which solution has real charactristic curve [3]. Such characteristic curve is nothing but the magnetic field line. Because of the nonlinearity, any two characteristic curves may intersect at any point and the field becomes discontinuous at that point.

III. METHODS TO DEMONSTRATE THE FORMATION OF MAGNETIC DISCONTINUITY

In order to demonstrate the formation of magnetic discontinuity in a fluid with large electrical conductivity, both analytical and numerical methods are adopted. There are basically two different methods by which formation of magnetic discontinuity can be demonstrated. First method is by shuffling the footpoints of magnetic field lines and then relaxing these shuffled field lines towards the equilibrium and searching for the possibility of the formation of MDs [3,4]. The second method is by deforming the domain of the magnetofluid and then doing the foot point mapping of field lines in both deformed and undeformed volumetric domain [5]. Parker uses the first approach to demonstrate the formation of magnetic discontinuity. In the Parker approach, the magnetic field lines are embedded in a fluid with large electrical conductivity. One end points of all these field lines are held fixed while at the other end points fluid motion is generated. Because of the motion of the fluid, the field lines will also move with the fluid due to the frozen-in condition. After some time, the magnetic field lines will get interwoven and form a complex magnetic topology. Once the field lines achieve such a complex field topology then the fluid motion is stopped. Now the two ends of each magnetic field lines are tied and the whole system is allowed to relax. Under the action of Maxwell stresses, the field lines acquire the terminal state of static equilibrium. In this equilibrium state the arbitrary topology of field lines do not match with the preferred topology of static equilibrium. However, the magnetic field will settle into the static equilibrium by forming the magnetic discontinuity. As all the analytical methods use simple sets of field topology in order to tackle the inherent nonlinear partial differential equations, resulting in a huge debate that whether MD forms as suggested by Parker or not.

Numerical methods adopt the technique of viscous relaxation in order to demonstrate the formation of MD [6,7]. In this method, the magnetic field is evolved by Navier-Stoke's equation and induction equation of magnetohydrodynamics. The magnetofluid evolves under the action of unbalanced forces such as Lorentz force, viscous drag, pressure, etc. In absence of any electrical resistivity, the magnetic energy gets converted into kinetic energy which is dissipated by viscosity. As the kinetic energy becomes zero, the magnetofluid achieves the terminal states of equilibrium. In this state, the large electrical conductivity of the magnetofluid prohibits any two fluid elements to intermix and as a result of that discontinuity in mgnetic field arises.

IV. POSSIBLE APPLICATIONS OF MAGNETIC DISCONTINUITY TO SOME REAL PHYSICAL SYSTEMS

Magnetic discontinuity appears in a wide range of physical systems such as solar corona, Earth's magnetosphere, laboratory plasmas, etc [1]. In the following we discuss the importance of the magnetic discontinuity in such physical systems. The outermost layer of the solar atmosphere is called the solar corona where the temperature is hovering in the the range of million degree Kelvin [3]. However, the temperature of the surface of the Sun, called the photosphere, is of the order of just few thousand Kelvin. The relatively high temperature of the solar corona contradicts the second law of thermodynamics that heat must flow from high temperature to a low temperature region. But in the case of the solar atmosphere, the heat flows from the corona to the photosphere. It turns out that there must be some balancing mechanism operating in the solar corona where energy is delivered from the source of energy, i.e. the photosphere. It is widely believed that the magnetic field of the Sun is responsible to transfer energy from the photosphere to the corona and constantly dissipating the magnetic energy in the corona at the location of magnetic discontinuity to maintain its temperature at million degree Kelvin temperature. Also, coronal medium has some finite electrical resistivity and this leads to reconnection of magnetic field lines. This process of magnetic reconnection generates various phenomena at the Sun such as solar flares, coronal mass ejections, etc.

As we know that the Earth has its own magnetic field. Due to the impact of solar wind, in magneto-tail region [8] of the Earth, the magnetic field lines are pinched and as a result of this when two topologically distinct region of field lines come into direct contact with each other then they form the magnetic discontinuity. This further leads to magnetic reconnection and fast energetic particles are produced which follows the field lines and may enter into the polar regions of the Earth to produce a beautiful colorful event called Aurora.

V. CONCLUSION

The dynamics of magnetic field plays a major role in generation of magnetic discontinuity in a fluid with large electrical conductivity. Such a highly conducting fluid has a remarkable property to maintain the frozen-in condition to a very good approximation at large length scale. However, this frozen-in condition breaks down at small length scale and magnetic field changes its topology through the process of magnetic reconnection. In this process magnetic energy gets converted into heat and kinetic energy. The theoretical understanding that how MD forms in a high Reynolds number is a fundamental problem of magnetohydrodynamics, which is still debated due to the lack of exact proof. Several analytical and numerical works were done which are in support as well as against the MD formation. Although, this theory of MD formation has profound applications to satisfactorily explain various phenomena of the solar corona, Earth's magnetosphere, etc.

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