Analysis of Mass Transfer with Variable Thermal Conductivity in Non-Newtonian Fluid

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***Abstract* - Magneto hydrodynamic flows of mass transfer due to combined effect of porosity and visco-elasticity with variable thermal conductivity over a non-isothermal stretching sheet have been investigated analytically. The effect of various physical parameters like visco-elastic parameter, thermal conductivity and Schmidt number are analyzed on concentration profiles.**

***Index Terms* - Magneto hydrodynamic flow, Thermal conductivity, visco-elasticity, porosity.**

INTRODUCTION

The analysis of boundary layer flow over a stretching sheet play an important role in many engineering processes, such as extrusion of polymer sheet, cooling of metallic sheet in cooling bath, manufacturing of plastic films, artificial fibres, and paper production etc. The study of momentum and heat transfer is found to be necessary for determining the quality of final products of such processes which is explained in detail by Karwe and Jaluria [1988, 1991] Sakiadis [1961a, 1961b] was the first amongst the others to study such problems by considering the boundary layer viscous fluid flow over a continuous solid surface moving with constant velocity. It was then extended to that of stretching of a boundary sheet with linear velocity by Crane [1970]. This work has subsequently attracted several researchers; Erickson.et.al [1966] extended this problem to the case in which the transverse velocity at the moving surface is non-zero. Tsou-et.al [1967], who investigated heat transfer effect of moving sheet with constant surface velocity and temperature. However, in reality most of the liquids used in industrial applications particularly in polymer processing applications are of non-Newtonian in nature. The non-Newtonian fluids are being considered more important and appropriate in technological applications in comparison with Newtonian fluids. In view of the importance of these applications, Rajagopal et.al [1984], have studied the flow behavior of visco-elastic fluid over a stretching sheet and gave an approximate solution for the flow. It is more appropriate to consider the non-Newtonian behavior of these fluids in the analysis of the boundary layer flow and heat transfer characteristics, because in industrial applications most of the fluids such as plastic films and artificial fibers are not strictly Newtonian. Considering the survey of literature in non-Newtonian fluid flow. Abel and Veena (1998) studied the visco-elastic fluid flow and heat transfer in a porous medium over a stretching sheet. Gupta and Sridar [1985] analyzed the effect of visco-elastic parameter on non-Newtonian flow through porous medium. Many researchers such as Anderson [1992,1995], Chakrabarthi and Gupta [1979], Sarpakaya.T [1961] have done their work on MHD visco-elastic fluid Flows.

In above all studies the physical properties of the fluid are assumed to be constant, but for liquid metals, it has been found that the thermal conductivity k varies with temperature in an approximately linear manner, which is also true in some polymer solutions in the class of Walter's liquid B [1994], and that leads to non-linearity in the boundary value problem of heat transfer. Prasad et.al [2000] analyzed the effect of momentum and heat transfer of visco elastic fluid flow over a non-isothermal stretching sheet assuming the thermal conductivity varying linearly with temperature. Motivated by all these investigations, we contemplate to study the MHD visco-elastic fluid flow over a stretching sheet in presence of variable thermal conductivity. Mass transfer characteristic is analyzed.

MATHEMATICAL FORMULATION

Consider a steady state two-dimensional incompressible visco-elastic laminar flow of a Walter's liquid B in porous media over a semi-infinite stretching sheet coinciding with the plane *y=0*. The flow is generated due to stretching of the sheet, caused by the simultaneous application of two equal and opposite forces along *x-axis*. Keeping the origin fixed, the sheet is then stretched with a speed varying linearly with the distance from the origin *x=0*. The flow field is then exposed under the influence of uniform transverse magnetic field in such way that the effect of the induced magnetic field is negligible (Anderson [1992]). Hence the basic boundary layer equation governing the flow, heat and mass transfer in presence of internal heat generation takes the form

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\gamma\frac{\partial^2 u}{\partial y^2}-k_0\left\{u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right\}-\frac{\sigma B_o^2}{\rho}u-\frac{\gamma}{k'}u \quad (2)$$

$$u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=D\frac{\partial^2 c}{\partial y^2} \quad (3)$$

Here, σ is the electrical conductivity, B_0 is the applied magnetic field, k_0 is the visco-elastic parameter of the Walter's liquid *B*. k' permeability of porous medium, *Q* is the volumetric rate of heat generation, *k* is the thermal conductivity and *D* is the diffusivity. The other quantities have their usual meanings.

The boundary conditions governing the flow are

$u=bx \quad v=0$

$C=C_\infty+A_2x^\lambda$ at y=0

$c\to c_\infty$ as $y\to\infty$ (4)

Here *u* and *v* are velocity components along *x* and *y* directions respectively. A_1, *B* and A_2 are arbitrary constants, which depend on the nature of the boundary surface. C_w, C_∞ are concentration of chemical spices on the boundary surface and concentration in the flow region far away from the boundary surface respectively. *x* is measured along the stretching sheet and *y* is normal to the surface.

FLOW ANALYSIS

In order to obtain the mathematical form of the velocity, we introduce the following similarity transformations

$$u=bxf'(\eta),\ \ v=-\sqrt{b\nu}\,f(\eta)$$

Where $\eta=\sqrt{\frac{b}{\nu}}.y$ (5)

With these changes of variables, equation (1) is identically satisfied and equation (2) is transformed into the following non-linear ordinary differential equation.

$$f'^2-ff''=f'''-k_1\{2f'f'''-ff''''-f''^2\}-Mnf'-k_2f' \quad (6)$$

Where $k_1=\frac{k_ob}{\gamma},\ Mn=\frac{\sigma B_o^2}{b\rho},\ k_2=\frac{\gamma}{k'b}$

are non-dimensional visco-elastic, Magnetic and porosity parameters respectively and the boundary condition takes the form

$f=0 \quad f'=1$ at $\eta=0$

$f'\to 0 \quad f''\to 0$ as $\eta\to\infty$ (7)

Where prime denotes differentiation w.r.t η. The exact solution of equation (6) corresponding to the boundary conditions (7) is obtained as

$$f=\frac{1}{\alpha}(1-e^{-\alpha\eta}),$$

Where $\alpha=\sqrt{\frac{1+Mn+k_2}{1-k_1}}$ (8)

The solutions for velocity field is obtained as

$$u=bxe^{-\alpha\eta}\ ,\quad v=-\sqrt{b\gamma}\ \frac{1-e^{-\alpha\eta}}{\alpha} \quad (9)$$

It is of some interest to note that our result (9) gives the result of Anderson [1995] in the limiting case of *k=0*

To solve equation (3) we assume

$$\phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty} \quad (10)$$

Using (10), equation (3) transforms to

$$\phi''+Scf\phi'-Sc\lambda f'\phi=0 \quad (11)$$

Where $Sc=\frac{\upsilon}{D}$ is the Schmidt number.

The boundary conditions become

$\phi=1$ at $\eta=0$

$\phi=0$ as $\eta\to 0$ (12)

Since equation (11) is ordinary linear differential equation, solving equation (11) subject to the boundary conditions (12) is obtained in the following form of confluent hyper-geometric function namely, Kummers function (M.Abromowitz and Stegun I.A [1972])

$$\phi(\eta) = \frac{e^{-\alpha\left(\frac{Sc}{\alpha^2}\right)\eta} F\left[\frac{Sc}{\alpha^2} - \lambda; \frac{Sc}{\alpha^2} + 1; \frac{-Sc}{\alpha^2} e^{-\alpha\eta}\right]}{F\left[\frac{Sc}{\alpha^2} - \lambda; \frac{Sc}{\alpha^2} + 1; \frac{-Sc}{\alpha^2}\right]} \quad (13)$$

RESULTS AND DISCUSSION

In order to have a clear insight of the physical problem, analytical results are displayed with the help of graphical illustration.

Table-I shows the value of wall concentration gradient $\phi'(0)$ for different values of *Sc* and k_1 . Fig-1 is drawn to display the graph of non-dimensional concentration profile $\phi(\eta)$ Vs.η for different values of Schmidt number *Sc* in porous and non-porous media.

We notice from this graph that the effect of increasing the values of *Sc* leads to decrease the concentration profile in the flow field. Physically, the increase of *Sc* means decrease of molecular diffusivity *D*, which results in decrease of concentration of boundary layer. Hence, the concentration of the species is higher for small values of *Sc* and lower for large values of *Sc*, i.e. species diffusion layer thickness is thinner for heavier particles (large values of *Sc*) than for lighter particles (smaller values of *Sc*). This result is in agreement with Abel. M.S et al.[2002]

SUMMARY

Magneto hydrodynamic flows with mass transfer due to combined effect of porosity and visco-elasticity with variable thermal conductivity over a non-isothermal stretching sheet have been investigated analytically. The effect of various physical parameters like visco-elastic parameter, thermal conductivity parameter and Schmidt number are analyzed on concentration profile. The specific conclusions derived from our study are summarized as follows-

1. The thickness of the concentration boundary layer decreases with increase the values of Schmidt number (*Sc*) in porous media.
2. The thickness of the concentration boundary layer decreases with increase the values of Schmidt number (*Sc*) in non- porous media with reduced magnitude.
3. Concentration Profile $\phi(\eta)$ are quantitatively more in porous media.

Table-I : Wall concentration gradient $\phi'(0)$ for different values *Sc* and k_1

Sc	k_1	$\phi'(0)$	
		k_2=0.0	$k2$=0.2
1.0	0.1	-0.606017	-0.586521
	0.2	-0.592565	-0.571693
	0.3	-0.575966	-0.554272
2.0	0.1	-0.958004	-0.936514
	0.2	-0.943022	-0.920367
	0.3	-0.925047	-0.901046
3.0	0.1	-1.22922	-1.2075
	0.2	-1.21409	-1.19109
	0.3	-1.19586	-1.17135

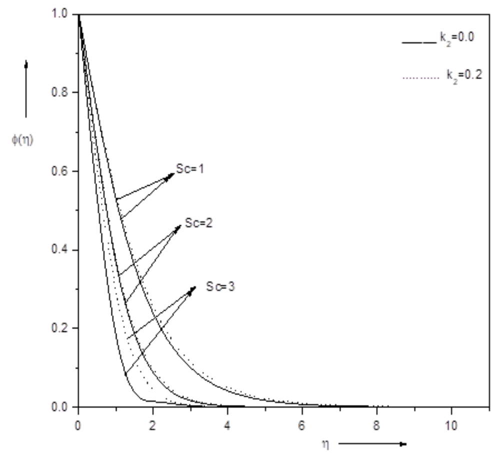

fig7:variation of concentration profile $\phi(\eta)$ Vs η for different values of Sc when k_1=.1, Mn=.1,λ=.1,ε=.1and Pr=1

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