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On Generalized Semi Closed Sets in Intuitionistic Topological Spaces

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***Abstract*- In this paper , we introduce a new class of sets namely , generalized semi closed sets in intuitionistic topological spaces . Also we have studied the concept of generalized semi connected spaces and discussed some properties in intuitionistic topological spaces.**

***Index Terms*- intuitionistic topological space, intuitionistic generalized semi closed sets, intuitionistic generalized semi connected space.**

I. INTRODUCTION

Generalized closed sets is a basis for the research in general topology. In 1970, Levine [7] introduced the concept of g-closed sets. Intuitionistic fuzzy sets was first introduced by Atanassov[1]. Later coker[2] introduced a new concept of intuitionistic topological spaces. Gnanambal Ilango[5] has given some results in intuitionistic sets and intuitionistic generalized pre regular closed sets in intuitionistic topological spaces. Duraisamy[4] studied some weakly open functions in intuitionistic topological spaces . The purpose of this paper , is to develop generalized semi-closed set in ITS. Further we have studied the concept of generalized semi connected spaces and discussed some properties in intuitionistic topological spaces.

2. PRELIMINARIES

We recall some definitions and properties which will be useful for our proceedings. Here , a space X means an intuitionistic topological space(X,τ).

Definition: 2.1[2] Let X be a non-empty set. An intuitionistic set (IS for short) A is an object having the form A=<X, A_1, A_2>, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \varphi$. The set A_1 is called the set of members of A , while A_2 is called the set of non-members of A.

Definition: 2.2 [2] Let X be a non-empty set and A, B be intuitionistic sets in the form A = < X, A_1, A_2 >, B = < X, B_1, B_2 > respectively. Then

(a) A ⊆ B iff $A_1 \subseteq B_1$ and $A_2 \supseteq B_2$.
(b) A = B iff A ⊆ B and B ⊇ A.
(c) Ā = < X, A_2 , A_1 >.
(d) [ ] A = < X, A_1 , $(A_1)^C$ >.
(e) A – B = A ∩ $\overline{B}$.
(f) ϕ = < X, φ, X >, X= < X, X, φ>.
(g) A ∪ B = < X, $A_1 \cup B_1$, $A_2 \cap B_2$ >.
(h) A ∩ B = < X, $A_1 \cap B_1$, $A_2 \cup B_2$ >.

Furthermore, let { A_i : i∈ J} be an arbitrary family of intuitionistic sets in X, where A_i = < X, $A_i^{(1)}$, $A_i^{(2)}$ >. Then

(i) ∩ A_i = < X, $\cap A_i^{(1)}$, $\cup A_i^{(2)}$ >.
(j) ∪ A_i = < X, $\cup A_i^{(1)}$, $\cap A_i^{(2)}$ >.

Remark: 2.3 [6] Any topological space (X,τ) is obviously an ITS of the form τ = { A' : A ∈ τ } where A' = < X, A, A^C >.

Definition: 2.4 [6] An intuitionistic topology (IT for short) on a non-empty set X is a family of IS's in X containing φ , X and closed under finite infima and arbitrary suprema. The pair(X,τ) is called an intuitionistic topological space (ITS for short). Any intuitionistic set in τ is known as an intuitionistic open set (IOS for short) in X and the complement of IOS is called intuitionistic closed set (ICS for short).

Definition: 2.5 [6] Let (X,τ) be an ITS and A = < X, A_1, A_2 > be an IS in X. Then the interior and closure of A are defined as

Icl(A)= ∩{K : K is an ICS in X and A ⊆ K}

Iint(A) = ∪{G : G is an IOS in X and G ⊆ A}.

It can be shown that Icl(A) is an ICS and Iint(A) is an IOS in X and A is an ICS in X iff Icl(A) = A and is an IOS in X iff Iint(A) = A.

Definition: 2.6 [2] Let X be a non-empty set and p∈ X. Then the IS defined by p= < X, {p}, $\{p\}^C$ >is called an intuitionistic point (IP for short) in X. The intuitionistic point is said to be contained in A = < X, A_1, A_2 > (i.e p ∈ A) if and only if p ∈ A_1 .

Definition: 2.7 [6] Let (X,τ) be an ITS. An intuitionistic set A of X is said to be

(i)intuitionistic semiopen if A ⊆Icl(Iint(A)).

(ii) intuitionistic preopen if A ⊆Iint(Icl(A)).

(iii) intuitionistic regular open (intuitionistic regular closed) if A = Iint(Icl(A)) (A = Icl(Iint(A))).

(iv)intuitionistic α-open if A⊆Iint(Icl(Iint(A))).

(v)intuitionistic b-open if A⊆Iint(Icl(A))∪Icl(Iint(A)).

The family of all intuitionistic preopen, intuitionistic regular open and intuitionistic α-open sets of (X, τ) are denoted by IPOS, IROS and IαOS respectively.

Definition: 2.8 [6] Let (X,τ) be an ITS and A = < X, A_1, A_2 > be an IS in X. Then

(i) the interiorand closure of an intuitionistic preopen set A is defined as

Ipcl(A) =∩{K: K is an IPCS in X and A⊆ K} ,

Ipint(A) = ∪{G : G is an IPOS in X and G⊆ A}.

(ii) the interior and closure of an intuitionistic α-open set A is defined as

Iαcl(A) =∩ {K : K is an IαCS in X and A⊆ K} ,

Iαint(A) = ∪{G : G is an IαOS in X and G⊆ A}.

Definition: 2.9 [5] Let (X,τ) be a non-empty ITS and let A = < X, A_1, A_2 > be IS. Then A is said to be

(i)intuitionistic generalized closed (Ig-closed) if Icl(A) ⊆ U whenever A ⊆ U and U is intuitionistic open in X.

(ii) intuitionistic generalized α-closed (Igα-closed) if Iαcl(A) ⊆ U whenever A⊆ U and U is intuitionistic open in X.

(iii)intuitionistic semi generalized closed (Isg-closed) if Iscl(A) ⊆ U whenever A⊆ U and U is intuitionistic semi open in X.

(iv)intuitionistic generalized semi regular closed (Igsr-closed) if Iscl(A) ⊆ U whenever A⊆ U and U is intuitionistic regular open in X.

(v) intuitionistic w-closed (Iw-closed) if Icl(A) ⊆ U whenever A ⊆ U and U is intuitionistic semi open in X.

(vi) intuitionistic g*-closed (Ig*-closed) if Icl(A) ⊆ U whenever A ⊆ U and U is intuitionistic generalised open in X.

Definition :2.10 [6] An intutionistic subset A of (X,τ) is said to be I-dense if cl(A)=X.

Definition : 2.11 [6] A space (X,τ) is called intuitionistic irreducible or I-hyperconnected if every intuitionistic open subset of X is I-dense.

3. INTUITIONISTIC GENERALIZED SEMI CLOSED SETS

Definition: 3.1 Let (X,τ) be an ITS and A=<X, A_1, A_2 > be an intuitionistic set. Then A is said to be intuitionistic generalized semi closed (Igs-closed) if Iscl(A) ⊆ U whenever A⊆ U and U is intuitionistic open in X . The family of all Igs - closed subsets of (X,τ) is denoted by IGSC(τ).

The complement of intuitionistic generalized semi closed sets are intuitionistic generalized semi open (Igs-open) and the family of all Igs - open subsets of (X,τ) is denoted by IGSO(τ).

Proposition: 3.2 Every I - closed set is Igs-closed.

Proof : Let A be an I-closed in an ITS(X,τ) and A⊆ U , where U is intuitionistic open .Since A is I closed , Icl(A) =A then, Iscl(A) ⊆ Icl(A) ⊆ U , Hence A is Igs-closed.

The converse of the above proposition is not true and is shown in the given example .

Example : 3.3 Let $X=\{a,b\}$ and $\tau = \{ X , \varphi , < X,\{a\},\phi> , < X,\{a\},\{b\}> , <X, \phi,\{b\}> \}$. Now the intuitionistic subset $<X, \phi,\{b\}>$ is Igs - closed but not I -closed .

Proposition :3.4 Every Ig-closed set is Igs-closed .

Proof : Let $A \subseteq X$ be Ig-closed in an ITS(X,τ) , then $Icl(A) \subseteq U$ whenever $A \subseteq U$ and U is intuitionistic open . Then $Iscl(A) \subseteq Icl(A) \subseteq U$. Hence A is Igs-closed.

The converse of the above proposition is not true and is shown in the following example.

Example : 3.5 Let $X=\{a,b\}$ and $\tau = \{ X , \varphi , < X,\{a\}, \phi > , < X,\{a\},\{b\}> , <X, \phi,\{b\}> \}$. Now the intuitionistic subset $<X \phi,\{b\}>$ is Igs – closed but not Ig closed.

Proposition : 3.6 Every Igα - closed set is Igs-closed.

Proof :Let $A\subseteq X$ be an Igα - closed set in an ITS(X,τ) and $A \subseteq U$ and U be an intuitionistic open , then $I\alpha cl(A) \subseteq U$. This implies $Iscl(A) \subseteq I\alpha cl(A) \subseteq U$ Hence A is Igs-closed.

The converse of the above proposition does not hold and is shown in the following example

Example : 3.7 Let $X=\{a,b\}$ and $\tau = \{ X , \varphi , < X,\{a\}, \phi > , < X,\{a\},\{b\}> , <X, \phi,\{b\}> \}$. Now the intuitionistic subset $<X, \phi,\{b\}>$ is Igs - closed but not Igα closed.

Proposition : 3.8 Every Iw-closed set is Igs-closed.

Proof: Let $A\subseteq X$ be an Iw-closed set in ITS(X,τ) , then $Icl(A) \subseteq U$ whenever $A \subseteq U$ and U be intuitionistic open then $Iscl(A) \subseteq Icl(A) \subseteq U$ Hence A is Igs-closed.

The converse of the above proposition is not true and is shown in the following example

Example : 3.9 Let $X=\{a,b\}$ and $\tau = \{ X , \varphi , < X,\{a\}, \phi > , < X,\{a\},\{b\}> , <X, \phi,\{b\}> \}$. Now the intuitionistic subset $<X, \phi,\{b\}>$ is Igs – closed but not Iw-closed.

Proposition : 3.10 Every Ig*- closed set is Igs-closed.

Proof : Let $A \subseteq X$ be an Ig*- closed set in ITS(X,τ) and $A\subseteq U$ where U is an intuitionistic g- open , since A is Ig*-closed , then $Icl(A) \subseteq U$ which implies $Iscl(A) \subseteq Icl(A) \subseteq U$ hence A is Igs-closed set.

The converse of the above proposition is not true and is shown in the following example

Example : 3.11 Let $X=\{a,b\}$ and $\tau = \{ X , \varphi , < X,\{a\}, \phi > , < X,\{a\},\{b\}> , <X, \phi,\{b\}> \}$. Now the intuitionistic subset $<X, \phi,\{b\}>$ is Igs – closed , but not Ig* closed.

Proposition :3.12 Every Igs- closed set is Igsr-closed.

Proof :Let $A \subseteq X$ be an Igs closed set in ITS(X,τ) and $A\subseteq U$ where U is an intuitionistic regular open , since every intuitionistic regular open is intuitionistic open , thus $Iscl(A)\subseteq U$. Hence A is Igsr-closed set.

The converse of the above proposition does not hold and is shown in the following example

Example :3.13 Let $X=\{a,b\}$ and $\tau = \{ X , \varphi , < X,\{a\}, \phi > , < X,\{a\},\{b\}> , <X, \phi,\{b\}> \}$. Now the intuitionistic subset $< X,\{a\},\{b\}>$ is Igsr-open but not Igs-open.

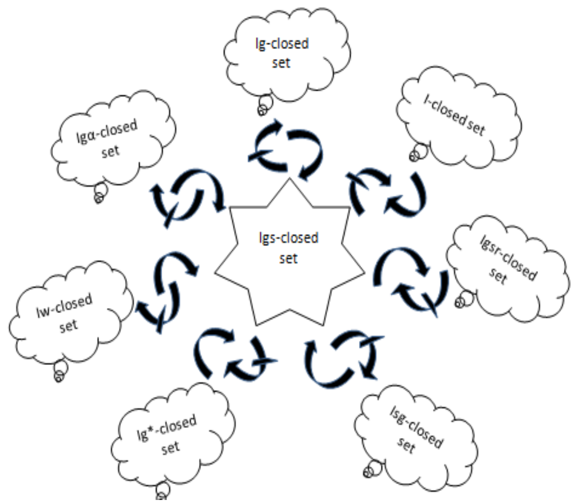
Proposition :3.14 Every Isg-closed is Igs-closed set.

Proof: Let $A \subseteq X$ be an Isg-closed set in ITS(X,τ) and $A\subseteq U$ where U is an intuitionistic open , since every intuitionistic open is intuitionistic semi open , thus $Iscl(A) \subseteq Icl(A) \subseteq U$. Hence A is Igs-closed set.

The converse of the above proposition does not hold and is shown in the following example

Example :3.15 Let X={a,b} and τ = { X , φ , < X,{a}, ϕ > , < X,{a},{b}> , <X, ϕ,{b}> }. Now the intuitionistic subset < X,{b}, ϕ > is Igs-open but not Isg-open.

Remark : 3.16 The diagrammatic representation is as follows



Theorem : 3.17 Let A and B be any two subsets of the intuitionistic topological space (X,τ) then Igs-cl (A∩B) ⊂Igs-cl(A) ∩ Igs-cl(B).

Proof: Since A∩B ⊂A,B , we have Igs-cl (A∩B) ⊂Igs-cl(A) and Igs-cl(A∩B) ⊂Igs-cl(B) , thus Igs-cl(A∩ B) ⊂Igs-cl(A) ∩ Igs-cl(B) .

Theorem : 3.18 If ISC(X,τ) be I closed under finite unions , then IGSC(X,τ) is I closed under finite unions .

Proof: If ISC(X,τ) be I closed under finite unions .Let A,B ϵ IGSC(X,τ) and A ∪ B⊂ U where U is intuitionistic open in X . Then A ⊂ U and B ⊂ U hence Iscl(A) ⊆ U and Iscl(B)⊆ U .This implies Iscl(A) ∪ Iscl(B) ⊂ U. Now Iscl(A) ⊂ U so A ∪ B ϵ IGSC(X,τ) .

Theorem: 3.19 Let A be an Igs-closed set of an intuitionistic topological space(X,τ) and A⊆ B ⊆ Iscl(A) then B is Igs - closed in X.

Proof: Let A be an Igs-closed set of an ITS(X,τ) and A ⊆ B ⊆Iscl(A). Let U be an intuitionistic open set such that B ⊆ U . Then A⊆ U and since A is Igs closed we have Iscl(A) ⊆ U . Now B ⊆Iscl(A) which implies Iscl(B) ⊆ Iscl (Iscl(A)) = Iscl(A) ⊆ U , hence B is Igs-closed in X.

Theorem :3.20 Let A be an intuitionistic subset of an intuitionistic topological space (X,τ) then A is Igs-open if and only if U ⊆ Isint(A) whenever U is intuitionistic open and U⊆A.

Proof :
Necessity : Let A be Igs-open in X and U be intuitionistic closed in X such that U⊆A , then U^C is intuitionistic open in X such that $U^C \subseteq A^C$, A^C is Igs-closed so Iscl(A^C) $\subseteq U^C$, but Iscl(A^C)=(Isint(A)$)^C \subseteq U^C$ which implies U ⊆ Isint(A).
Sufficiency : Let F be an intuitionistic open in X such that A^C ⊆F. Then F^C is intuitionistic closed in X and F^C ⊆ A.
To prove : A^C is Igs-closed
Now $F^C \subseteq$ Isint(A) which implies Iscl(A^C) = (Isint(A)$)^C$ ⊆ F , hence A^C is Igs-closed which implies A is Igs-open in X.

Theorem :3.21 Let A be an intuitionistic generalized open set of an intuitionistic topological space (X,τ) and Isint(A) ⊆ B ⊆ A then B is Igs open.

Proof : Now Isint(A) ⊆ B ⊆A. since (Isint(A) $)^C$ = Iscl(A^C), $A^C \subseteq B^C \subseteq$ Iscl(A^C) then A^C is Igs closed by theorem 3.18 which implies B^C is also Igs-closed then obviously B is Igs-open.

4. INTUITIONISTIC GENERALIZED SEMI CONNECTED SPACE

Definition: 4.1 Let (X,τ) be an intuitionistic topological space. Then X is called Intuitionistic generalized semi – connected (I-connected, Ig-connected, Igα-connected, Isg-connected, Ig*-connected) , if there does not exists an proper intuitionistic set (φ ≠ A≠ X) of X which is both intuitionistic generalized semi-open(I-open,Ig-open,Igα-open,Isg-open,Ig*
open)and intuitionistic generalized semi closed (I-closed ,Ig-closed ,Igα-closed ,Isg-closed ,Ig*-closed).

Proposition: 4.2 Every Igs - connected space is intuitionistic connected.

Proof: Let (X,τ) be an Igs - connected space and not intuitionistic connected. Then there exists a proper intuitionistic subset of X which is both intuitionistic openand intuitionistic closed . Every intuitionistic open set and intutitionistic closed set is Igs - open and Igs-closed , then X is not Igs-connected which is a contradiction , Therefore X is Igs-connected.

Proposition: 4.3 Every Igs-connected space is Ig-connected.

Proof: Let (X,τ) be an Igs-connected space and suppose that not (X,τ) is Ig-connected. Then there exists a proper intuitionistic set of X which is both Ig-open and Ig-closed . we know every Ig-open and Ig-closed is Igs-open and Igs-closed ,then X is not Igs-connected which is a contradiction.

Proposition: 4.4 Every Igs-connected space is Igα-connected.

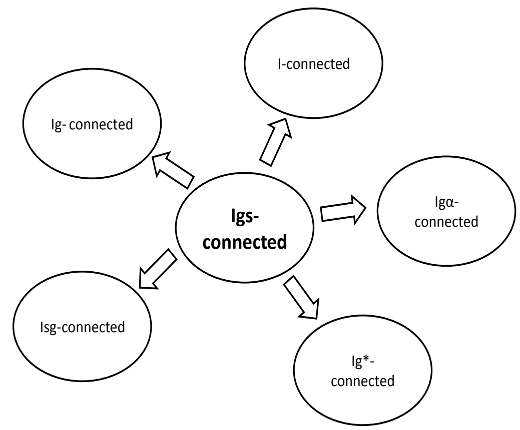
Proof: Let (X,τ) be an Igs-connected space and suppose that not (X,τ) is Igα-connected. Then there exists a proper intuitionistic set of X which is both Igα-open and Igα-closed . we know every Igα-open and Igα-closed is Igs-open and Igs-closed ,then X is not Igs-connected which is a contradiction.

Proposition :4.5 Every Igs-connected space is Isg-connected.

Proof : Let (X,τ) be an Igs-connected space and suppose that not (X,τ) is Isg-connected. Then there exists a proper intuitionistic set of X which is both Isg-open and Isg-closed . we know every Isg-open and Isg-closed is Igs-open and Igs-closed ,then X is not Igs-connected which is a contradiction.

Proposition :4.6 Every Igs-connected space is Ig*-connected.
Proof: Let (X,τ) be an Igs-connected space and suppose that not (X,τ) is Ig*-connected. Then there exists a proper intuitionistic set of X which is both Ig*-open and Ig*-closed . we know every Ig*-open and Ig*-closed is Igs-open and Igs-closed ,then X is not Igs-connected which is a contradiction.

Remark: 4.7 The diagrammatic representation is as follows



Theorem : 4.8 For an intuitionistic topological space (X,τ) , if X is I-hyperconnected then every intuitionistic subset of X is Igs-closed.

Proof : If X is I-hyperconnected ,then the only intuitionistic open subsets of X are $< X, \varphi, X >$ and $< X, X, \varphi >$. So every intuitionistic subset of X is Igs-closed.

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